

Saint Vrain Valley School District
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Ten Plus One:

Enhancing Depth and Complexity of
Math Tasks (K – 2)

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Following along today:

[5280math.com/ten-plus-one/](https://www.5280math.com/ten-plus-one/)

Following up after today:

(1) I will get a copy of these slides to you.

(2) My website has supporting materials.

<http://www.5280math.com/ten-plus-one/>

(3) See a post on today's topic at MiddleWeb:

Ten Tweaks That Can Deepen Math Tasks

<https://www.middleweb.com/36542/10-tweaks-that-can-deepen-math-tasks/>

Goals

- *Experience* the look and feel **depth** and **complexity** in math.
- *Learn* and *practice* using strategies to enhance depth and complexity of routine math tasks.
- *Envision* implementing enhanced tasks in your classroom.
- *Anticipate* your students' mathematical thinking.

Mathematical Depth

ROUTINE THINKING

FACTS

remember

PROCEDURES

practice

PERFORMANCE

accuracy and speed

Actions

DEEP THINKING

PATTERNS

observe and analyze

RELATIONSHIPS

explore

Goals

UNDERSTANDING

meaning

To learn more, see *Mathematical Mindsets*, by Jo Boaler.

Ten Strategies

for creating deep math tasks

5280math.com >> [5280 Math Resources](#) >> [Ten Plus One](#)

1. Write a story.
2. Draw a picture.
3. Explain why.
4. Find another way.
5. Compare and contrast.
6. Start with the answer.
7. Remove information.
8. Solve to learn.
9. Build a pattern.
10. Ask “What if...?”

One Strategy

for creating complex math tasks

Use *more...*

digits, numbers, shapes, parts, variety, steps, ideas,
information, definitions, categories, *relationships*

Caution: The purpose is not just messy computation.

Common Core Connections

Task

Write the total amount of money.



1.NBT.B.2

Understand that the two digits of a two-digit number represent amounts of tens and ones.

2.MD.C.8

Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately.
Example: If you have 2 dimes and 3 pennies, how many cents do you have?

The Ten Plus One Process

for making math tasks deeper and more complex

1. **Identify** a routine math task.
2. **Choose** a depth strategy.
3. **Create** a deeper task.
4. **Adjust** the complexity (optional).
5. **Anticipate** students' thinking.

1. Identify a math task.

5. Anticipate students' thinking

2. Choose a depth strategy.

3. Create a deeper task.

4. Adjust the complexity (optional).

1. Identify a math task.

Write the value of the coins.



2. Choose a depth strategy.

#1 Write a story.

3. Create a deeper task.

Write a story about this much money.



4. Adjust the complexity (optional).

Write a story about this much money
and some other amount.

(Show 3 dimes and 6 pennies mixed up.)

5. Anticipate students' thinking

Students' stories might not involve determining the total value. If not, ask them to revise the stories so that they do.

Some students' stories may involve thinking of real-world things that could have a value of 36¢ (not much these days!). If so, ask them to compare stories and talk about this.

For the more complex problem, I wonder if students will create stories that involve comparing their two amounts (or doing something else with them).

Benefits of teaching with tasks created from Ten Plus One

The tasks make your students' thinking visible.

The tasks help you build instruction
around your students' ideas.

1. Identify a math task.

Write the value of the coins.



2. Choose a depth strategy.

#2 Draw a picture.

#5 Compare and contrast.

3. Create a deeper task.

Draw a picture that helps you compare the value of each set of coins.

(Show two sets: 3 dimes and 6 pennies;
2 dimes and 11 pennies.)

4. Adjust the complexity (optional).

The deeper task is already more complex, because it involves two sets of coins. (though I could use more coins / types of coins).

5. Anticipate students' thinking

Some students may show replacing dimes by 10 pennies.

Others may draw abstract representations of coins as something else (tally marks, dots, etc.).

Students choose their own amount for the second set of coins. Hopefully, they will choose an appropriate level of challenge. If not, let them start with what they choose. Then talk about their experience, and steer them in another direction.

Some “pictures” may actually be numbers organized in a way to show counting strategies. This is okay.

1. Identify a math task.

Write the value of the coins.



2. Choose a depth strategy.

#3 Explain why.

3. Create a deeper task.

Explain why the digits 3 and 6 in 36¢ are the same as the numbers of dimes and pennies.

(Show 3 dimes and 6 pennies.)

4. Adjust the complexity (optional).

Explain why the digits 2, 3, and 6 in 236¢ are the same as the numbers of dollar bills, dimes, and pennies.

(Show 2 dollar bills, 3 dimes, 6 pennies.)

5. Anticipate students' thinking

Students may struggle with understanding the sentence. We may need to discuss it verbally. Maybe this discussion will help me think of a better way to phrase the question.

This may be hard for students to put into words. I may try interviewing some students if it's too much for them to write.

The final goal is to get them to make explicit connections to place value using the appropriate language.

Name : _____ date: _____ Concept: _____

Modeling Math Meaningfully

I Can write it with numbers!

I Can draw a picture of it.

I Can write a story problem.

I Can model it using _____ math tools and explain my thinking

Created by: Jennifer Suh

<http://mason.gmu.edu/~jsuh4/pictures/figure3.modeling%20math.JPG>

1. Identify a math task.

Write the value of the coins.



2. Choose a depth strategy.

#4 Find another way.

#3 Explain why.

3. Create a deeper task.

Find two more ways to make this amount using only dimes and pennies. Explain how you found your answers.

(Show the 3 dimes and 6 pennies.)

4. Adjust the complexity (optional).

Make 57¢ in as many ways as possible using only dimes and pennies. Explain how you know you have found all possible ways.

5. Anticipate students' thinking

Some students may trade between 1 dime and 10 pennies.

Some may use different counting strategies: counting all pennies, counting dimes first, counting pennies first, etc.

In the more complex problem, students are likely to find and write answers randomly. This is a good chance to help them begin learning the value of organizing their data in a table and writing their answers according to a pattern.

1. Identify a math task.

Write the value of the coins.



2. Choose a depth strategy.

#6 Start with the answer.

#5 Compare and contrast.

#9 Build a pattern.

3. Create a deeper task.

Find 2 or more ways to make 36¢ with dimes and pennies. Compare your answers and describe patterns.

4. Adjust the complexity (optional).

Find all possible ways to make 36¢ with dimes and pennies. Choose other amounts and repeat. Compare your answers and describe patterns.

5. Anticipate students' thinking

In the more complex problem, students are likely to find and write answers randomly. This is a good chance to help them begin learning the value of organizing their data (say in a table) and writing their answers according to a pattern.

When they choose other amounts, talk to them about why they chose what they did. If they can't verbalize a reason, encourage them eventually to start trying to think of reasons for their choices.

One goal is to discover that all amounts in 30s will have 4 solutions. In general, the number of solutions will be 1 greater than the tens digit.

1. Identify a math task.

Write the value of the coins.



2. Choose a depth strategy.

#7 Remove information.
#3 Explain why.

3. Create a deeper task.

Some money is covered up. How much money do you think there is? Why?



4. Adjust the complexity (optional).

5. Anticipate students' thinking

I'm not sure if this is a great task, but I'd like to see what students do with it. There is a lot of room for creativity! They may:

- Try to keep a pattern going and stick with dimes and pennies.
- Guess how many coins and what kind would fit.
- Get very creative and imagine other types of coins, piles of coins, etc.
- Try to find many possible answers.
- Figure out new values by adjusting old ones instead of starting over in their counting.

1. Identify a math task.

Write the value of the coins.



2. Choose a depth strategy.

#8 Solve to learn.

3. Create a deeper task.

Keep the task the same, but give it before instructing. Possibly combine with other strategies such as #4 (find more than one strategy to count them) or #3 (explain).

4. Adjust the complexity (optional).

Still offer no instruction, but use greater numbers of dimes and pennies: especially (1) more than \$1.00 total, or (2) more than 10 pennies and/or dimes.

5. Anticipate students' thinking

Usually, I would make sure that students have the background knowledge first—like knowing the values of dimes and pennies.

If they don't: I may let them try it anyway, but I will have to (1) be prepared for creative guesses and ask them to explain them, or (2) allow them to do some research first.

If they do: I will probably have to use the more complex version of the problem with a goal of having them come up with creative strategies for counting larger amounts of money.

1. Identify a math task.

Write the value of the coins.



2. Choose a depth strategy.

#6 Start with the answer.

#10 Ask “What if....?”

#5 Compare and contrast.

#9 Build a pattern.

3. Create a deeper task.

How many ways can you make 36¢ with dimes and pennies? What if you have nickels and pennies instead? Compare your answers and look for patterns. Can you predict one answer from the other?

4. Adjust the complexity (optional).

Use larger total values of coins, possibly greater than \$1.00.

5. Anticipate students' thinking

This is a high-level thinking task—more like a project. Students are likely to struggle with choosing amounts, writing their ideas down, organizing their data, and looking for patterns, but these are important things for them to begin learning, and I think can they make progress.

I will let them work together, and I'll be ready to offer guidance without telling them what to do.

After they have worked for a while, I may give them a blank table they can use to record their results. I will guide some discussion so that they can learn from each other.

Sample tasks to enhance

grade
K

There are some counters on the page.
Emily puts 3 more on the page.
There are now 7 counters in all.



Topic 6 Lesson 5 page 314

You read a take-away story starting with 7 and
having a difference of 4. Students draw
counters, mark Xs, and write an equation.



Topic 7 Lesson 5 page 392

Draw another group of bugs so that there are
20 bugs in all.



20



Topic 9 Lesson 6 page 545

Write the numbers to continue each
pattern. Use a number chart to help.

grade
1

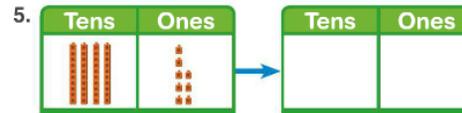
6. Count by 1s.

102, 103, 104, _____, _____

(7 more blanks)

Topic 7 Lesson 4 page 415

Count the tens and ones.
Then write the numbers.



_____ tens and _____ ones is _____.

Topic 8 Lesson 4 page 469

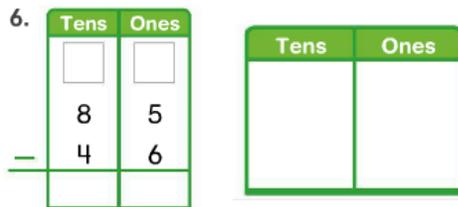
Write >, <, or =.

6. 34 ○ 43

Topic 9 Lesson 4 page 517

grade
2

Subtract. Draw place value blocks
to show work. Regroup if needed.



Topic 6 Lesson 4 page 343

Solve the problem any way you
choose. Show your work.

4. 25 wolves howl together in the woods.
14 wolves join them. Then 22 wolves run
away. How many wolves are left?

Topic 7 Lesson 5 page 417

Use place value blocks to show the
hundreds, tens, and ones. Then show
two other ways to make the number.

418 = _____

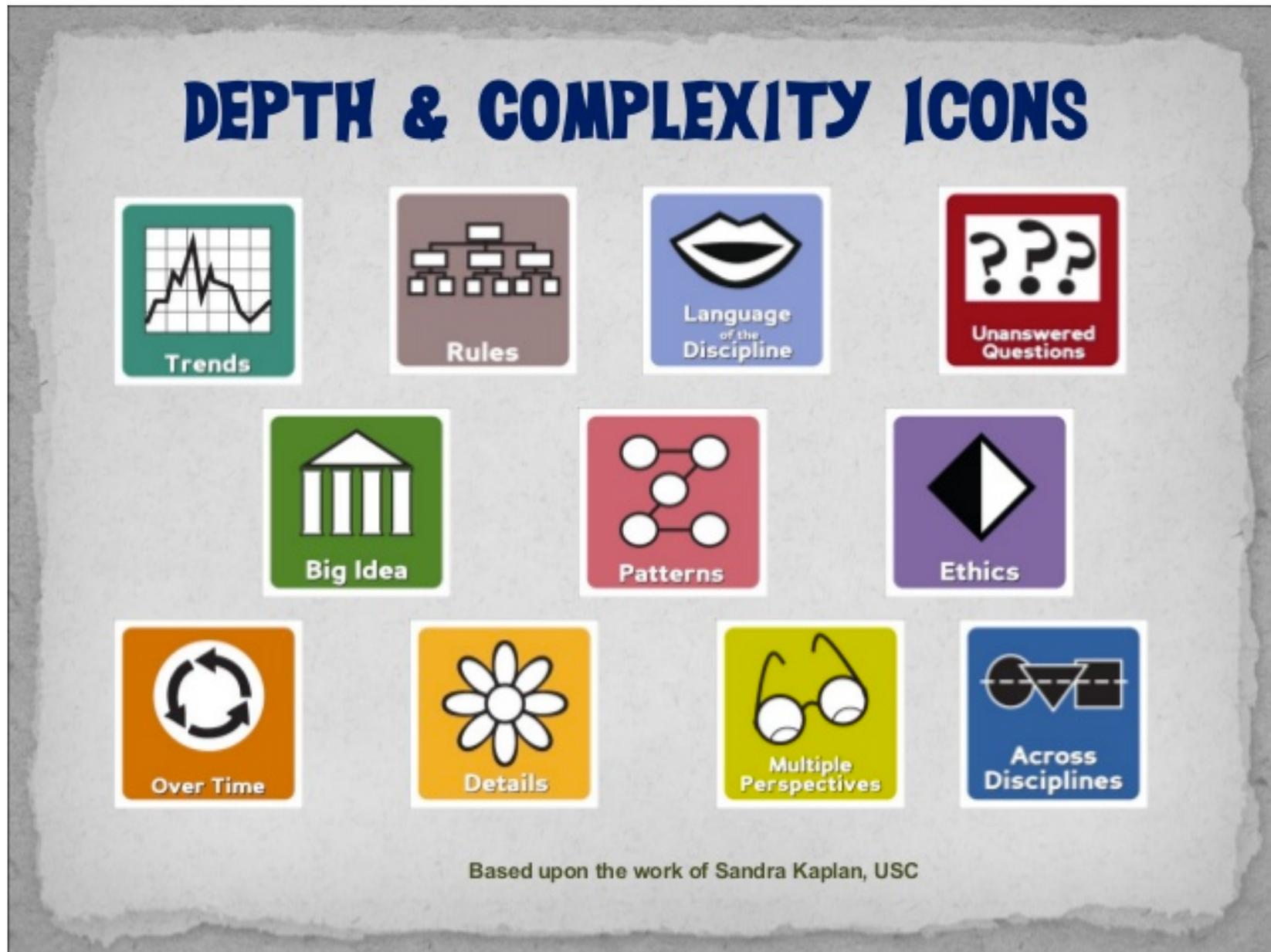
418 = _____

418 = _____

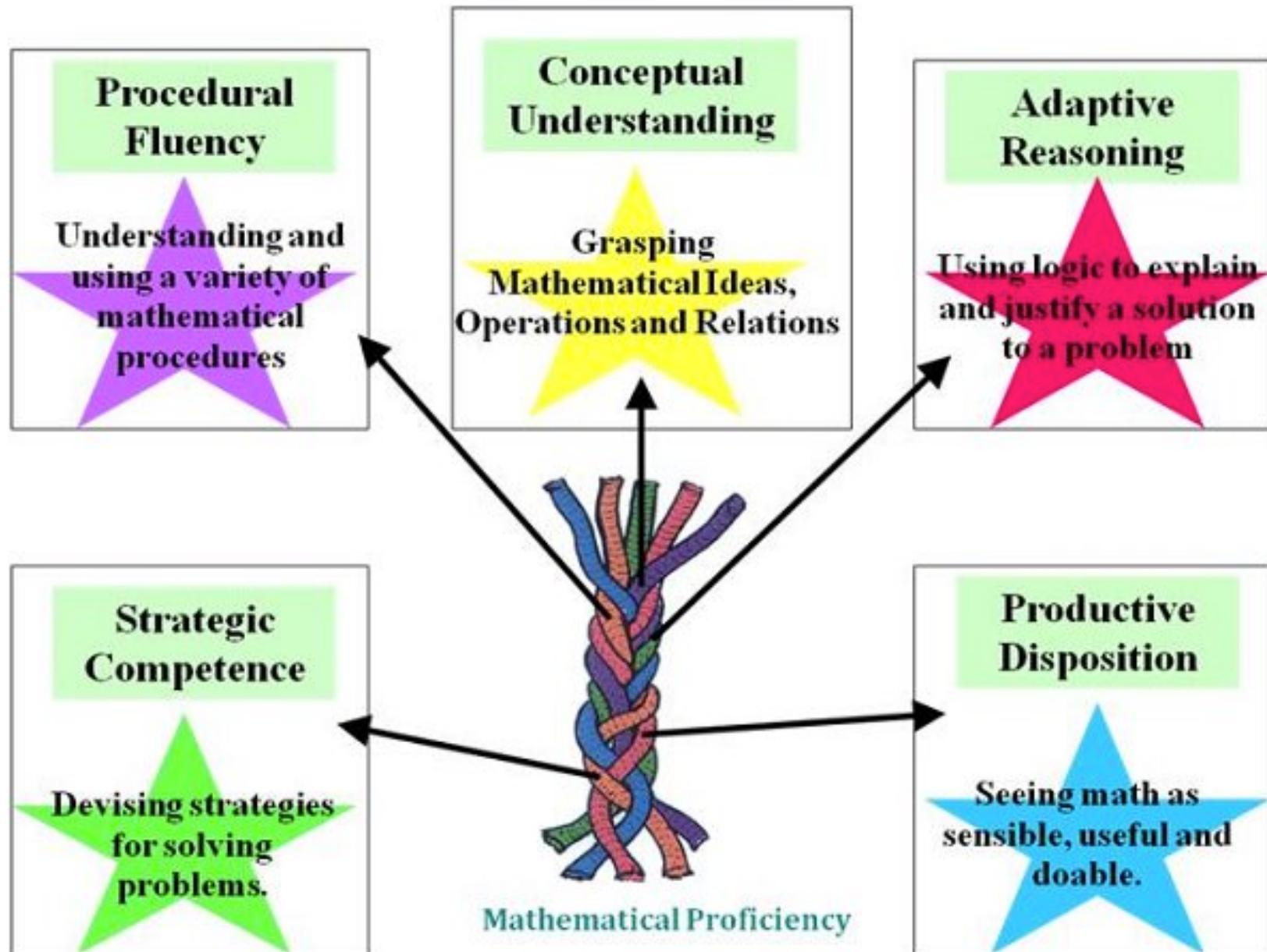
Topic 9 Lesson 5 page 537

Take a moment to think and talk about how you could use the tasks you have created.

Kaplan's Icons



Math Proficiency Strands



Kilpatrick, J., Swafford, J., Findell, B. (Ed.). (2001). Adding it up: helping children learn mathematics. Washington, DC: National Academy Press.

NCTM Process Standards

Problem Solving

Reasoning and Proof

Communication

Connections

Representations

Connecting Best Practices in Math and Gifted

Conceptual Understanding	Big Idea , Patterns, Trends, Different Perspectives
Procedural Fluency	Rules , Ethics, Different Perspectives
Adaptive Reasoning	Patterns and Details , Trends, Different Perspectives
Strategic Competence	Unanswered Questions , Different Perspectives, Ethics
Mathematical Communication	Language of the Discipline , Rules, Different Perspectives
Connections	Across the Disciplines , Different Perspectives, Patterns
Representations	Different Perspectives , Language of the Discipline

Creating New Tasks

Combine strategies

Mix and match.

Skip strategies

If a strategy doesn't work, let it go.

Be flexible

Let the strategies inspire your creativity!

Focus on concepts

Build on ideas *underlying* the task.

Take the long view

Keep creating new tasks and
improve them over time.

Using Your New Tasks

Step back

Let students do the thinking.

Allow collaboration

Let students learn from each other.

Expect explanations

Deep, complex ideas are worth expressing!

Learn as you go

Refine tasks using your students' ideas!

Save work samples

Use student work for exemplars and assessments.

Resources

from 5280 Math

- Ten Plus One from the 5280 Math website
- How Open-Ended Math Tasks Keep on Giving
from a middleweb.com post
- Noticing and Wondering: Pathways to Mathematical Meaning
from an edcircuit.com post
- Creative Math Prompts
- Advanced Common Core Math Explorations book series

Other Resources

- *Mathematical Mindsets* by Jo Boaler
- *Extending the Challenge in Mathematics* by Linda Jensen Sheffield
- *Good Questions: Great Ways to Differentiate Math Instruction*, by Marian Small
- *Which One Doesn't Belong?*, by Christopher Danielson



Supporting each other's work

Share tasks, students' ideas, and/or classroom experiences.

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