*CAGT, October 2017*

# Ten Plus One:

## Enhancing Depth and Complexity of Math Tasks

Jerry Burkhart
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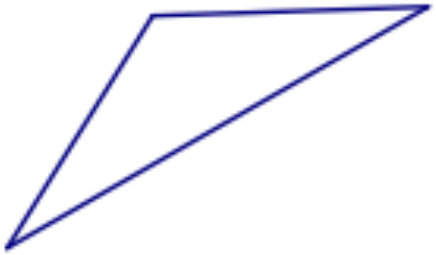
5280math.com

74 = ____ tens and ____ ones

35 – 18

Name the shape.



Simplify $\frac{18}{21}$.

Find the mean and median.

31, 27, 32, 65, 29

12 x 10

Measure the angle.

3.4 x 9.8

35% of 120

Write 0.027 in scientific notation.

Graph 3x + 2y = 5

Solve. $\sqrt{x-3} = 2x - 7$

# Mathematical Depth

**Traditional thinking**

What do I *do*?

What are the *steps*?

How can I *remember*?

**Deeper thinking**

What do I *think*?

What does it *mean*?

How does it *connect?*

**Deep** math is creative math!

# Goals

Learn strategies for creating deep math tasks.

Apply strategies for creating deep math tasks.

Anticipate students' thinking.

Envision classroom implementation.

Discuss additional resources.

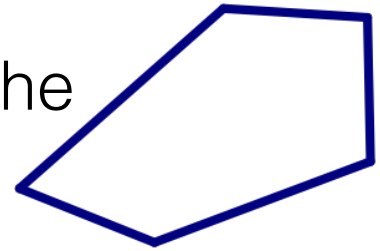
# Ten Strategies

## for creating deep math tasks

5280math.com >> 5280 Math Resources >> Ten Plus One

1. Write a story.
2. Draw a picture.
3. Explain why.
4. Find another way.
5. Compare and contrast.
6. Start with the answer.
7. Remove information.
8. Solve to learn.
9. Build a pattern.
10. Ask “What if…?”

# 1. Write a story.

| | |
|---|---|
| $36 + 7$ | Write >, <, or =.<br>0.7 ____ 0.58 |
| Name the shape.  | $6 \div \frac{2}{3}$ |
| $10 \times 37$ | $13 - -5$ |
| $352 \div 6$ | Find the slope of the line through the points (4, 7) and (6, 4). |

Write a story.

# One Strategy

## for creating complex math tasks

Use more…

digits, numbers, shapes, parts, variety, steps, ideas,

information, definitions, categories, relationships, etc.

**Caution: The purpose is not just messy computation.**

Write a story.

Use more…

# 2. Draw a picture.
# 3. Explain why.

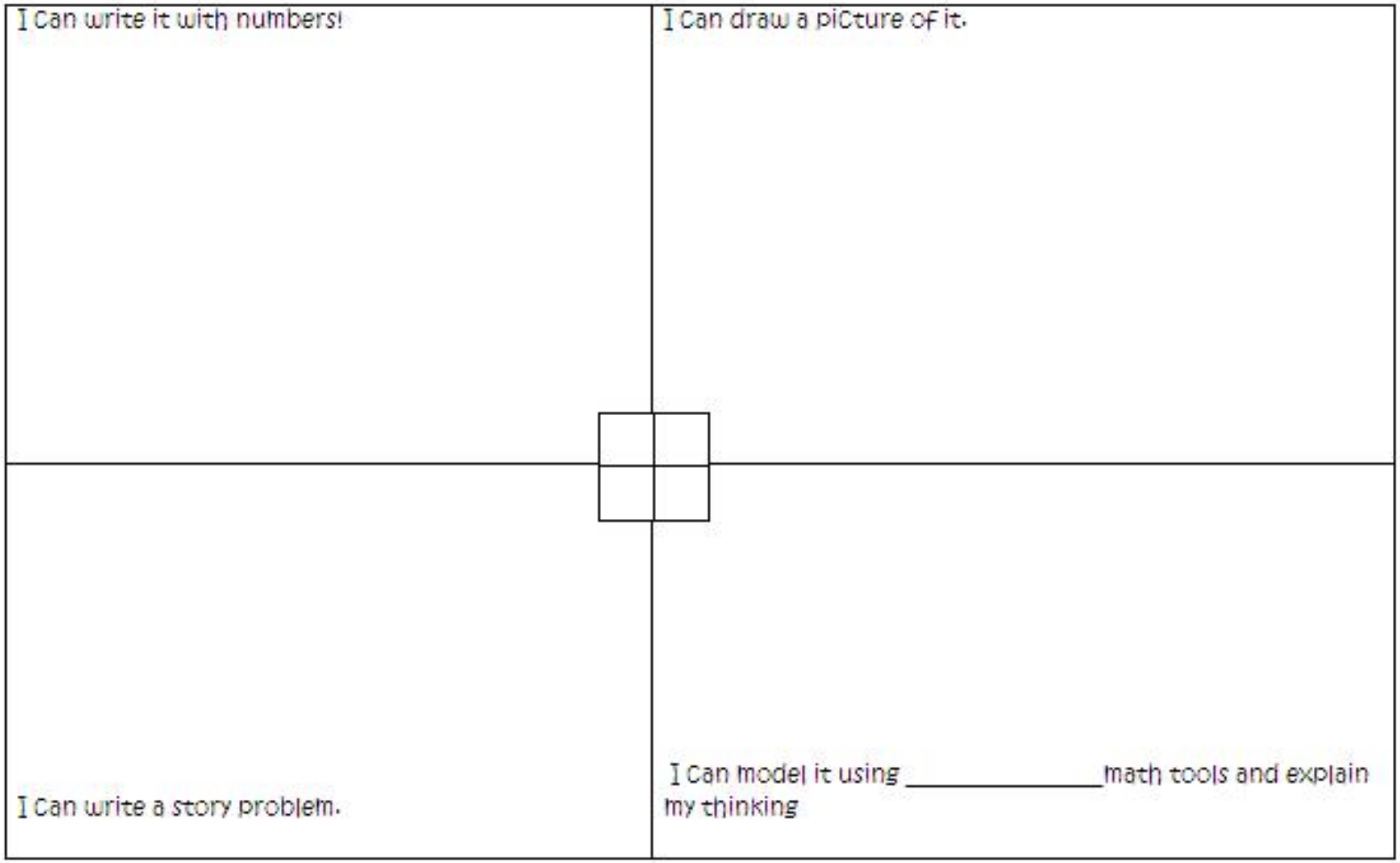
| | |
|---|---|
| $7 + 4 = ___ + 5$ | $6 \cdot \frac{2}{3}$ |
| $35 - 18$ | $6 \div \frac{2}{3}$ |
| $\frac{2}{3} = \frac{6}{\square}$ | $13 - -5$ |
| $62 \div 6$ | Find the hypotenuse of a right triangle having legs of length 3 cm and 4 cm. |

Write a story.
Draw a picture.
Explain why.

Use more…

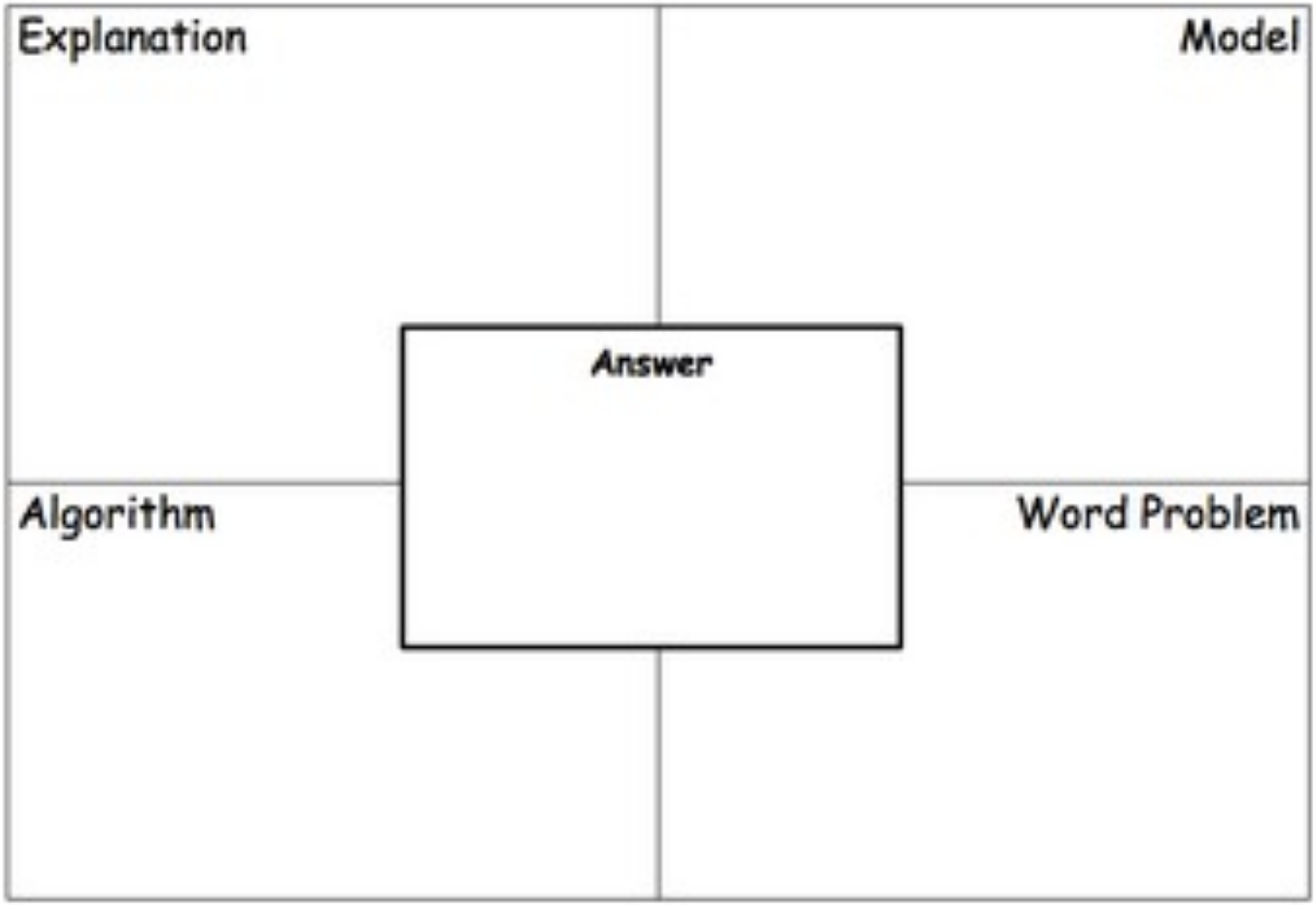
Name : _____ date: _____ Concept: _____

# Modeling Math Meaningfully




Created by: Jennifer Suh

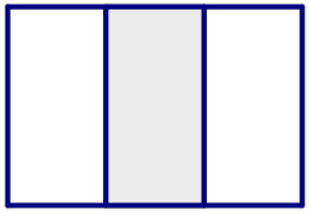
http://mason.gmu.edu/~jsuh4/pictures/figure3.modeling%20math.JPG




# Lesh Model

https://ecdn.teacherspayteachers.com/thumbitem/Math-Lesh-Model-Template-1418693-1409583324/original-1418693-1.jpg

# 4. Find another way.

| | |
|---|---|
| Find the value of the coins.<br>(A picture shows<br>3 dimes and 6 pennies.) | 8.4 x 1000 |
| What fraction does this show?<br>  | 18 is what<br>percent of 40? |
| 10 x 37 | A shirt that costs $16.50<br>is on sale for 20% off.<br>Find the sale price. |
| 62 ÷ 6 | Find the value of $3^{-2}$. |

Write a story.
Draw a picture.
Explain why.
Find another way.

Use more…

# The **Ten Plus One** Process

1. **Identify** a math task.
2. **Choose** a Ten Plus One *depth* strategy.
3. **Apply** the strategy to enhance the task.
4. **Adjust** the *complexity*.
5. **Anticipate** students' thinking.

# A Ten Plus One template

1. Identify a math task.

2. Choose a depth strategy.

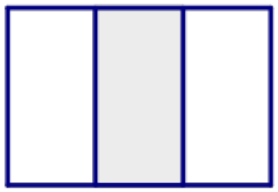
3. Apply the strategy.

4. Adjust the complexity.

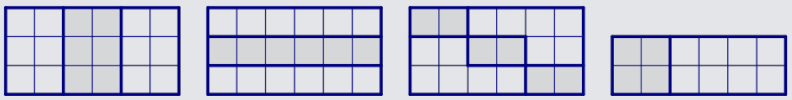
5. Anticipate students' thinking.

Jerry Burkhart 5280math.com

1. Identify a math task.
What fraction does the shaded part show?

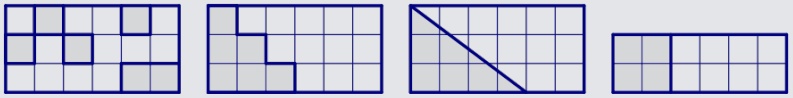


2. Choose a depth strategy.
#5 Compare and contrast.



3. Apply the strategy.
What do the first three pictures have in common? What do they all have in common?

4. Modify the complexity.
Use the same questions with these pictures.



5. Anticipate students' thinking.

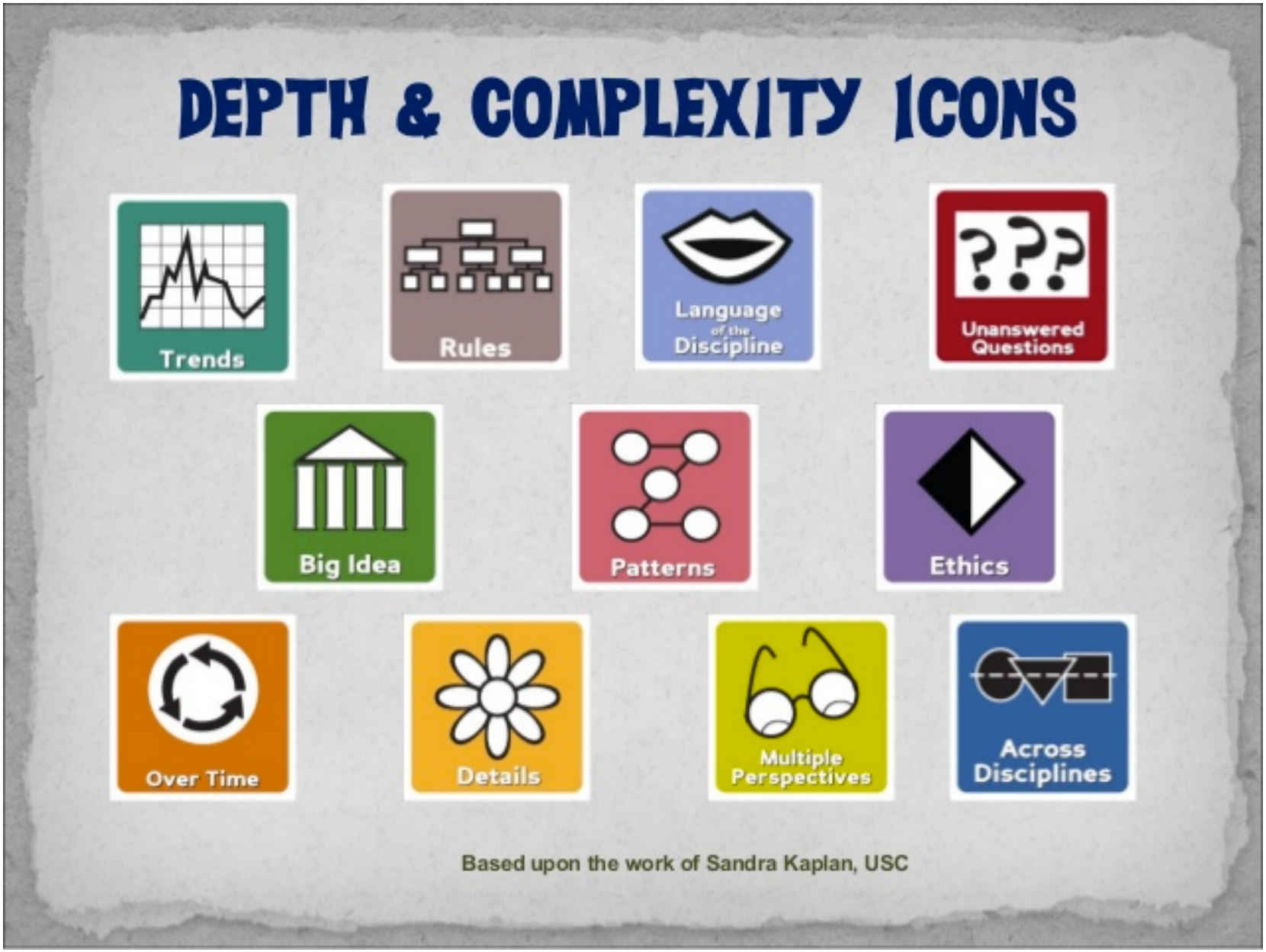
Some students may count squares and notice that the first three pictures have six shaded squares, but the last has only four.

In order to find something that the last picture has in common with the first three, students may need to compare the shaded part to the entire rectangle and notice that three copies of the shaded part fit into the whole thing.

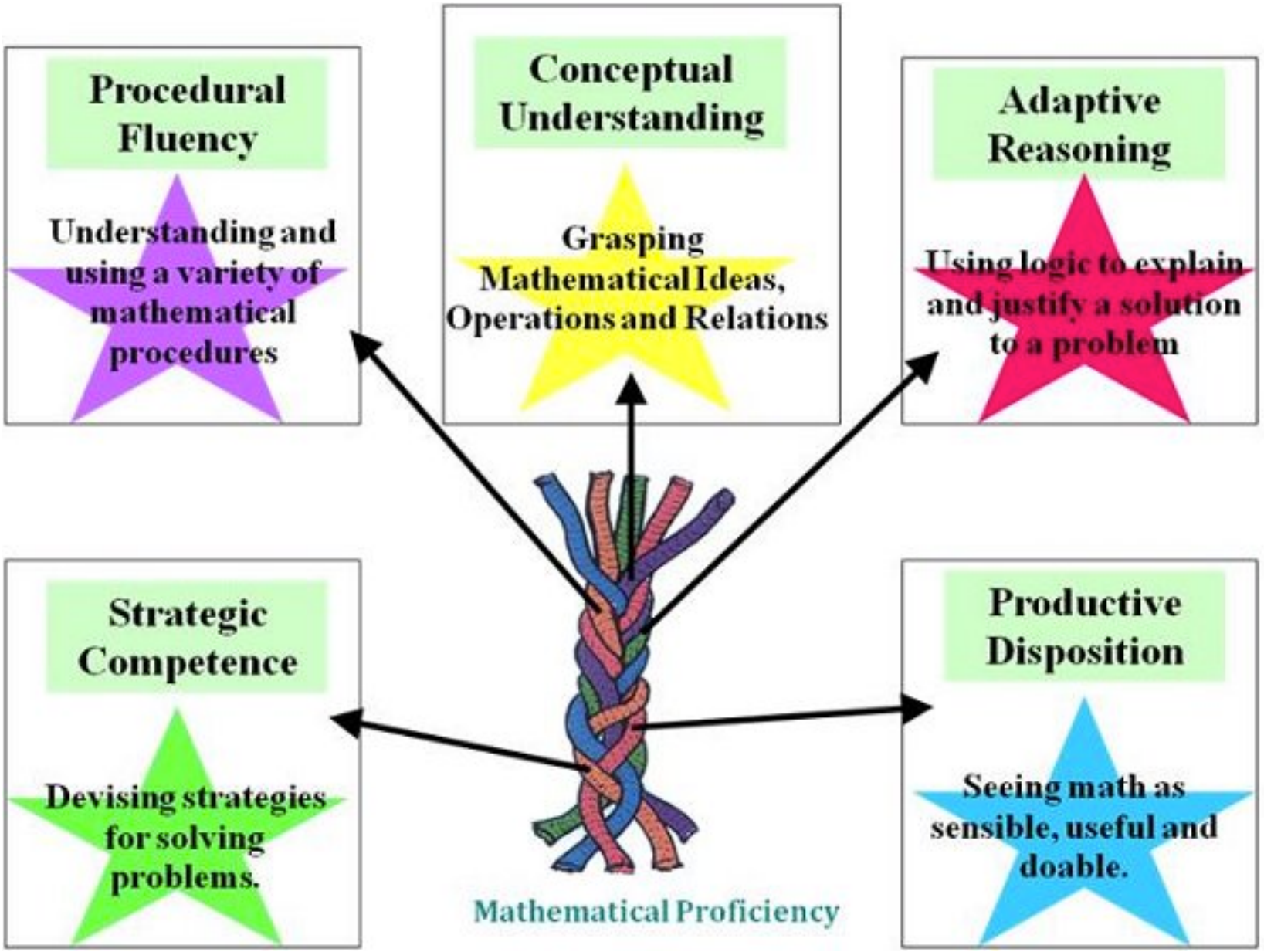
This may be harder for them to see in the third picture, because the pieces have to be rearranged.

The complex pictures require more imagination. In the first picture, the parts are not all connected. The third picture is more challenging because it includes parts of squares that are not as easily rearranged. Some students may notice that two copies of the triangle make a rectangle with twelve squares.

# Kaplan's Icons



# Math Proficiency Strands




Kilpatrick, J., Swafford, J., Findell, B. (Ed.). (2001). Adding it up: helping children learn mathematics. Washington, DC: National Academy Press.

# NCTM Process Standards

Problem Solving

Reasoning and Proof

Communication

Connections

Representations

adapted from *Principles and Standards for School Mathematics*. Reston, Va.: NCTM, 2000.

# Connecting Best Practices in Math and Gifted

| | |
|---|---|
| Conceptual Understanding | **Big Idea**, Patterns, Trends, Different Perspectives |
| Procedural Fluency | **Rules**, Ethics, Different Perspectives |
| Adaptive Reasoning | **Patterns** and **Details**, Trends, Different Perspectives |
| Strategic Competence | **Unanswered Questions**, Different Perspectives, Ethics |
| Mathematical Communication | **Language of the Discipline**, Rules, Different Perspectives |
| Connections | **Across the Disciplines**, Different Perspectives, Patterns |
| Representations | **Different Perspectives**, Language of the Discipline |

# Ten Strategies

## for creating deep math tasks

5280math.com >> 5280 Math Resources >> Ten Plus One

1. Write a story.
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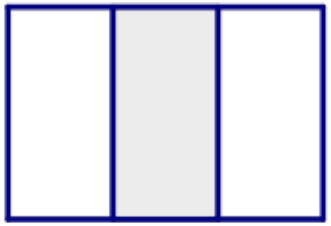
8. Solve to learn.
9. Build a pattern.
10. Ask “What if…?”

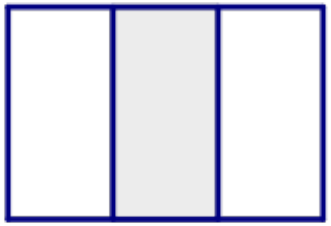
# 5. Compare and Contrast.

Task: What fraction is the shaded part?

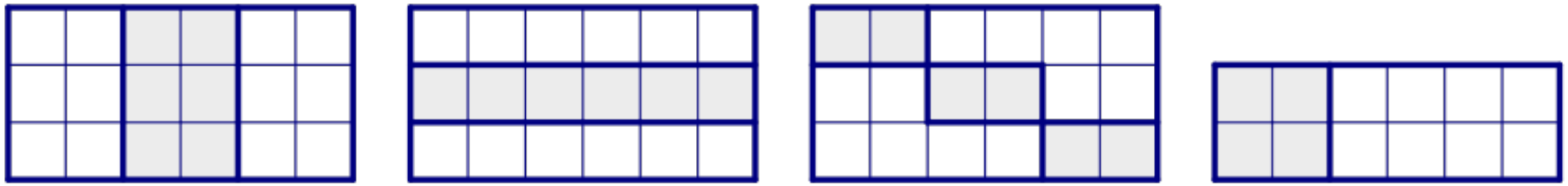


# 5. Compare and Contrast.

Task: What fraction is the shaded part?



New task: What do the pictures haves in common?



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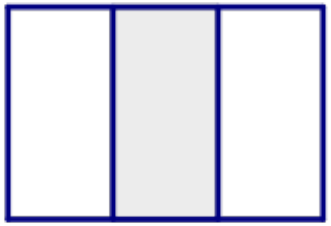

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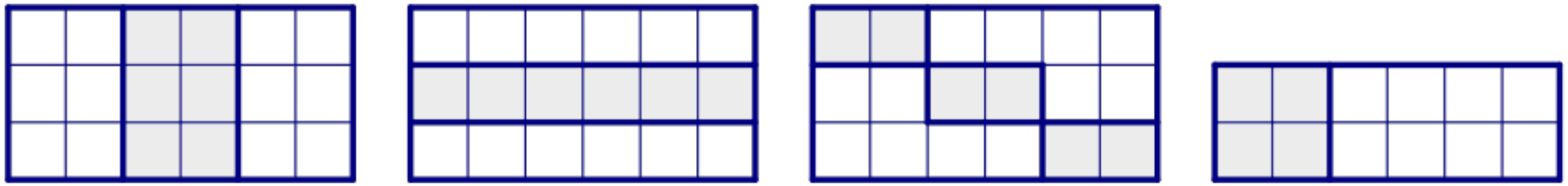
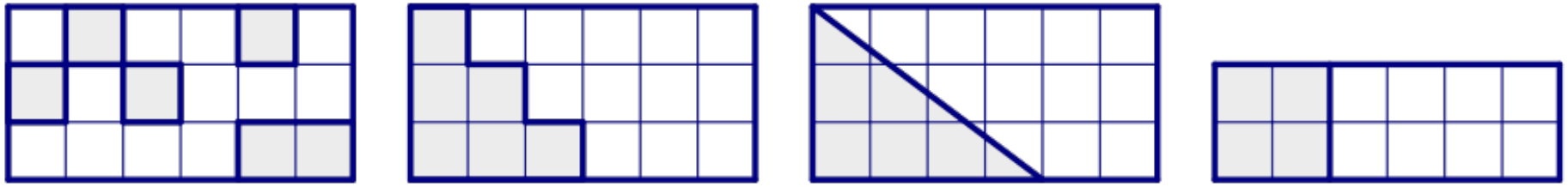


New Task+: What do the pictures have in common?



Write a story.
Draw a picture.
Explain why.
Find another way.
Compare and contrast.

Use more…

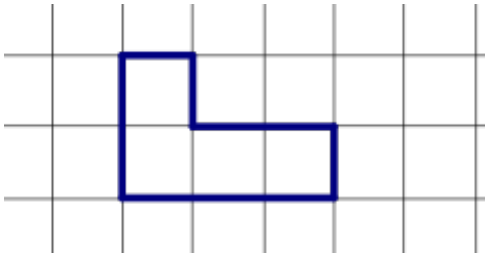
Math adventurers
are **curious!**

Math adventurers
are **risk-takers!**

Math adventurers
are **flexible thinkers!**

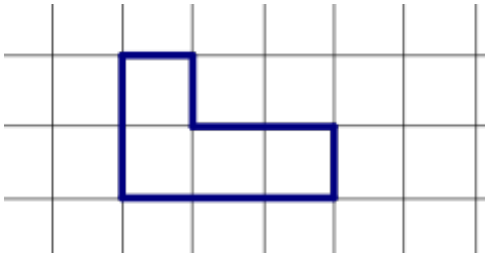
# 6. Start with the answer.

Task: Find the area.



# 6. Start with the answer.

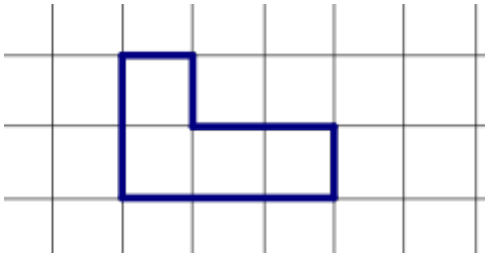
Task: Find the area.



New Task: Draw some polygons that have an area of 4 square units.

# 6. Start with the answer.

Task: Find the area.



New Task: Draw some polygons that have an area of 4 square units.

New Task+: The area is four and the perimeter is ___. Draw some polygons.

Write a story.
Draw a picture.
Explain why.
Find another way.
Compare and contrast.
Start with the answer.

Use more…

# 7. Remove information.

Task:

**36 ÷ 9 = ____**

# 7. Remove information.

Task:

**36 ÷ 9 = _____**

New Task:

**36 ÷ _____ = _____**

# 7. Remove information.

Task:

**36 ÷ 9 = ____**

New Task:

**36 ÷ ____ = ____**

New Task+:

**168 ÷ ____ = ____**

or

**36 ÷ ____ ÷ = ____ ÷ ____**

# 7. Remove information.

Task:

$$\frac{9}{b} = \frac{2}{6}$$

# 7. Remove information.

Task:

$$\frac{9}{b} = \frac{2}{6}$$

New Task: Find some solutions.

$$\frac{9}{b} = \frac{c}{6}$$

# 7. Remove information.

Task:

$$\frac{9}{b} = \frac{2}{6}$$

New Task: Find some solutions.

$$\frac{9}{b} = \frac{c}{6}$$

New Task+: Find ten solutions.

$$\frac{5}{b} = \frac{c}{3}$$

# **Math Adventurers** want to know!

- Why (or why not)?
- What if...?
- Is that *always* true?
- Is that *ever* true?
- How do I know?
- Is there another *answer?*
- Is there another *way* to think about it?
- Can I find a pattern?
- Does that make sense?

Write a story.
Draw a picture.
Explain why.
Find another way.
Compare and contrast.
Start with the answer.
Remove information.

Use more…

# Ten Strategies

## for creating deep math tasks

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7. Remove information.
8. Solve to learn.
9. Build a pattern.
10. Ask “What if…?”

**Choose task/strategy. Apply. Adjust complexity. Anticipate.**

# 8. Solve to learn.

Task:

$$\frac{1}{2}+\frac{1}{3}$$

# 8. Solve to learn.

Task:

$$\frac{1}{2}+\frac{1}{3}$$

New Task:

$$\frac{1}{2}+\frac{1}{3}$$

# Creating New Tasks

| | |
|---|---|
| Combine strategies | Mix and match. |
| Skip strategies | If a strategy doesn't work, let it go. |
| Be flexible | Let the strategies inspire your creativity! |
| Focus on concepts | Build on ideas underlying the task. |
| Take the long view | Improve your tasks over time. |

# Using Your New Tasks

| | |
|---|---|
| Step back | Let students do the thinking. |
| Allow collaboration | Students learn from each other. |
| Expect explanations | Deep, complex ideas are worth expressing! |
| Learn as you go | Refine tasks using your students' ideas! |
| Save work samples | Use student work for exemplars and assessments. |

# Mathematical Depth

**Traditional thinking**

What do I *do*?

What are the *steps*?

How can I *remember*?

**Deeper thinking**

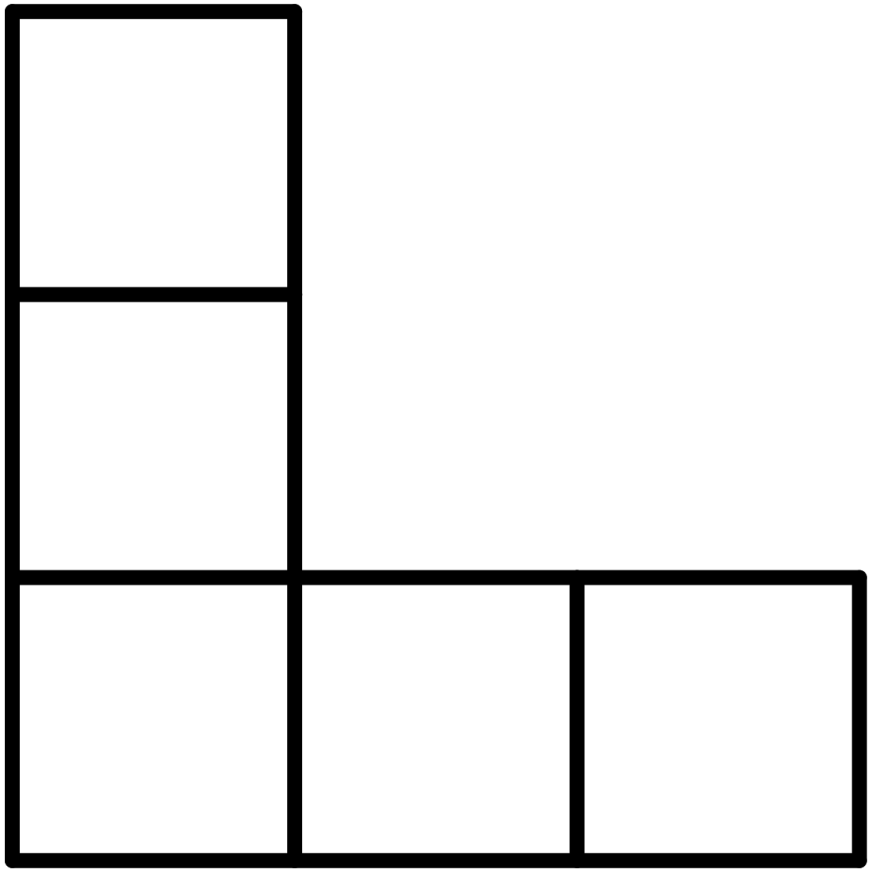
What do I *think*?

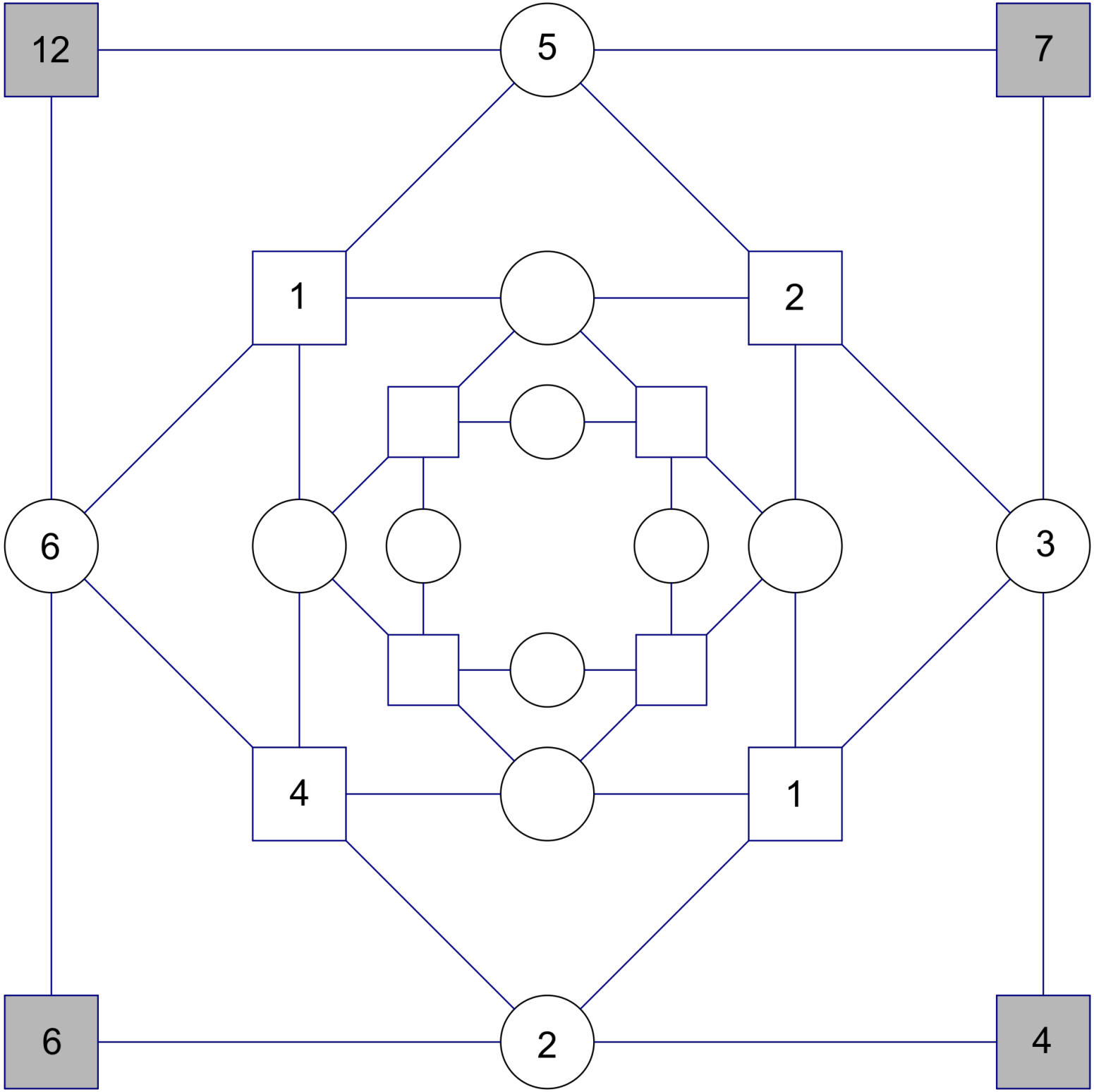
What does it *mean*?

How does it *connect?*

**Deep** math is creative math!

1 2 3 4 5




12
5
7
1
2
6
3
4
1
6
2
4

| · | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 |
| 9 | 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 |
| 10 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

614 - 416

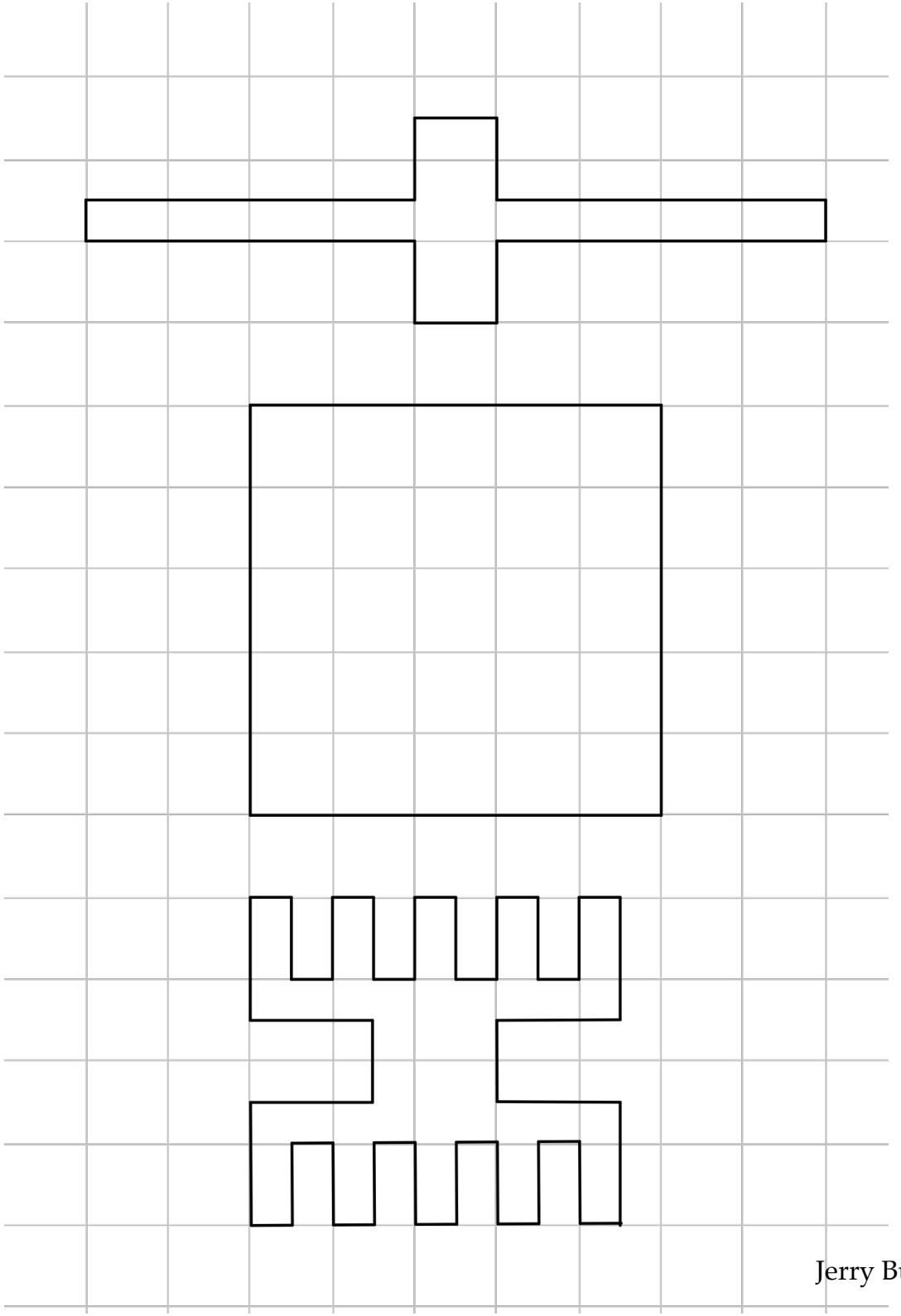
793 - 397

921 - 129

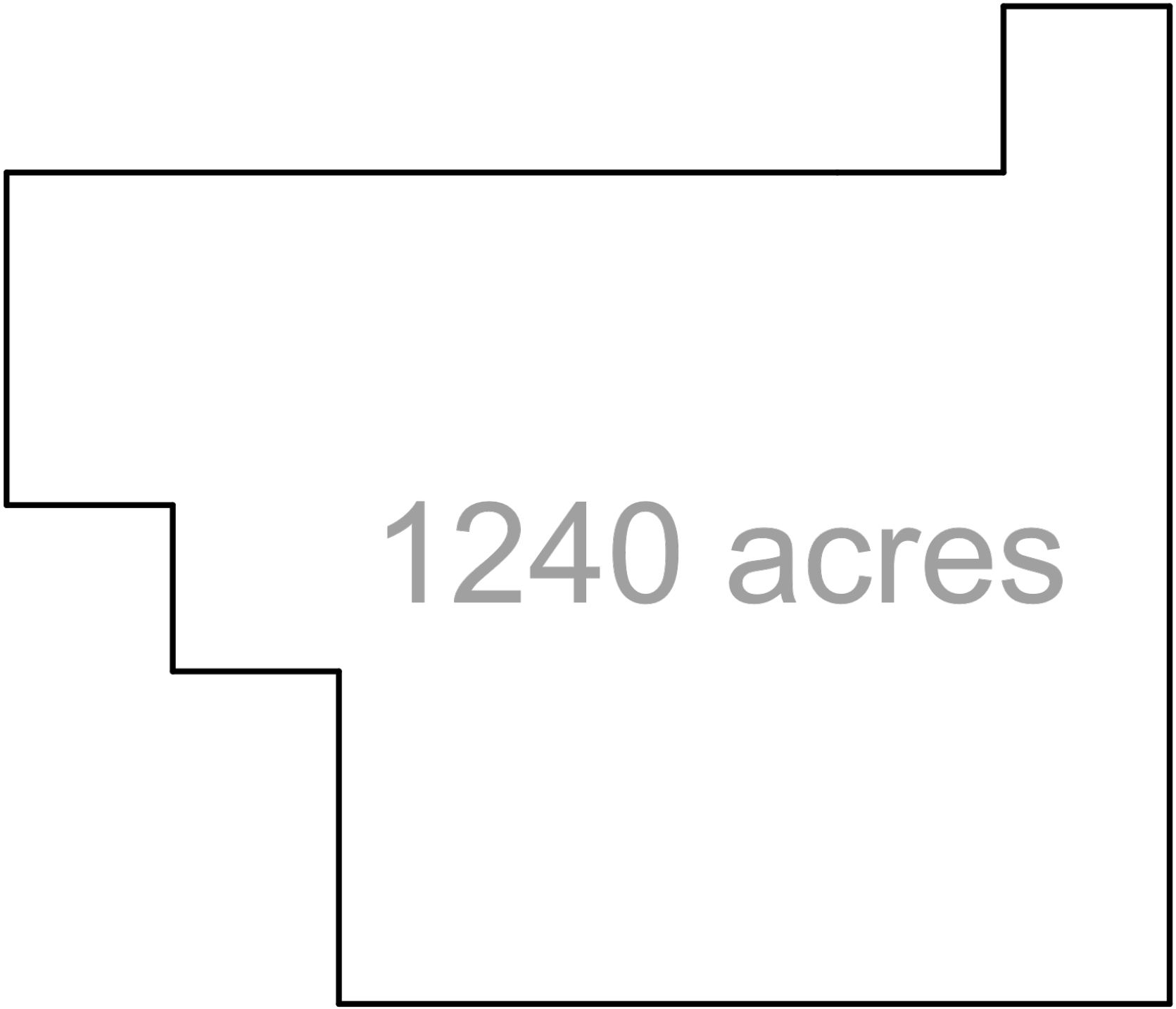
532 - 235

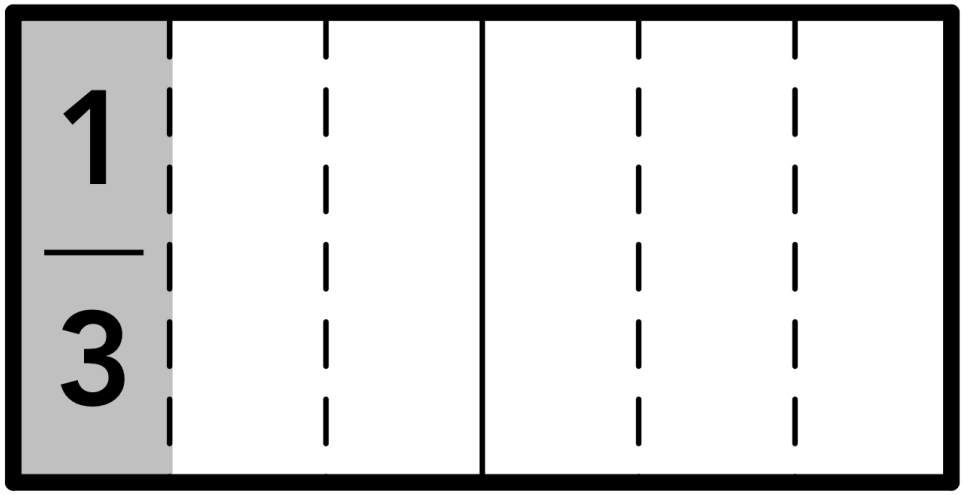
783 - 387

902 - 209



$$\frac{3}{5} \quad \frac{11}{18} \quad \frac{8}{13} \quad \frac{13}{21} \quad \frac{5}{8}$$


1240 acres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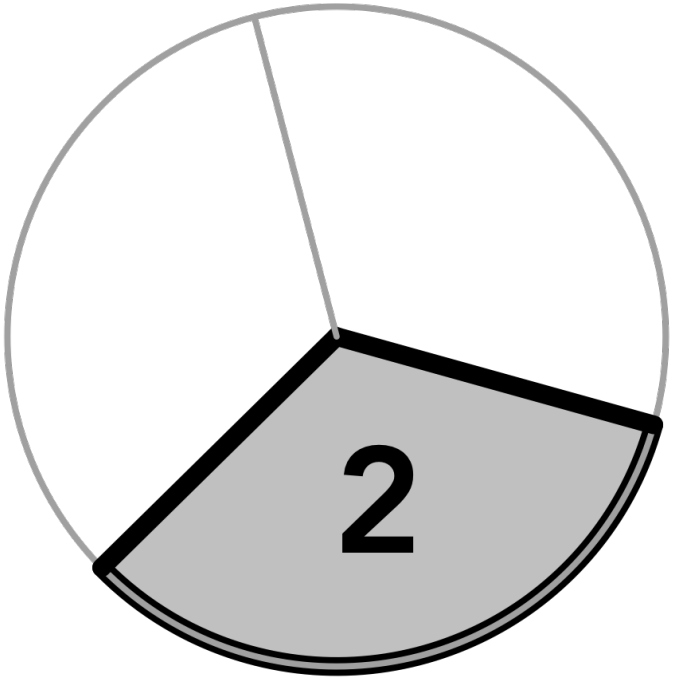
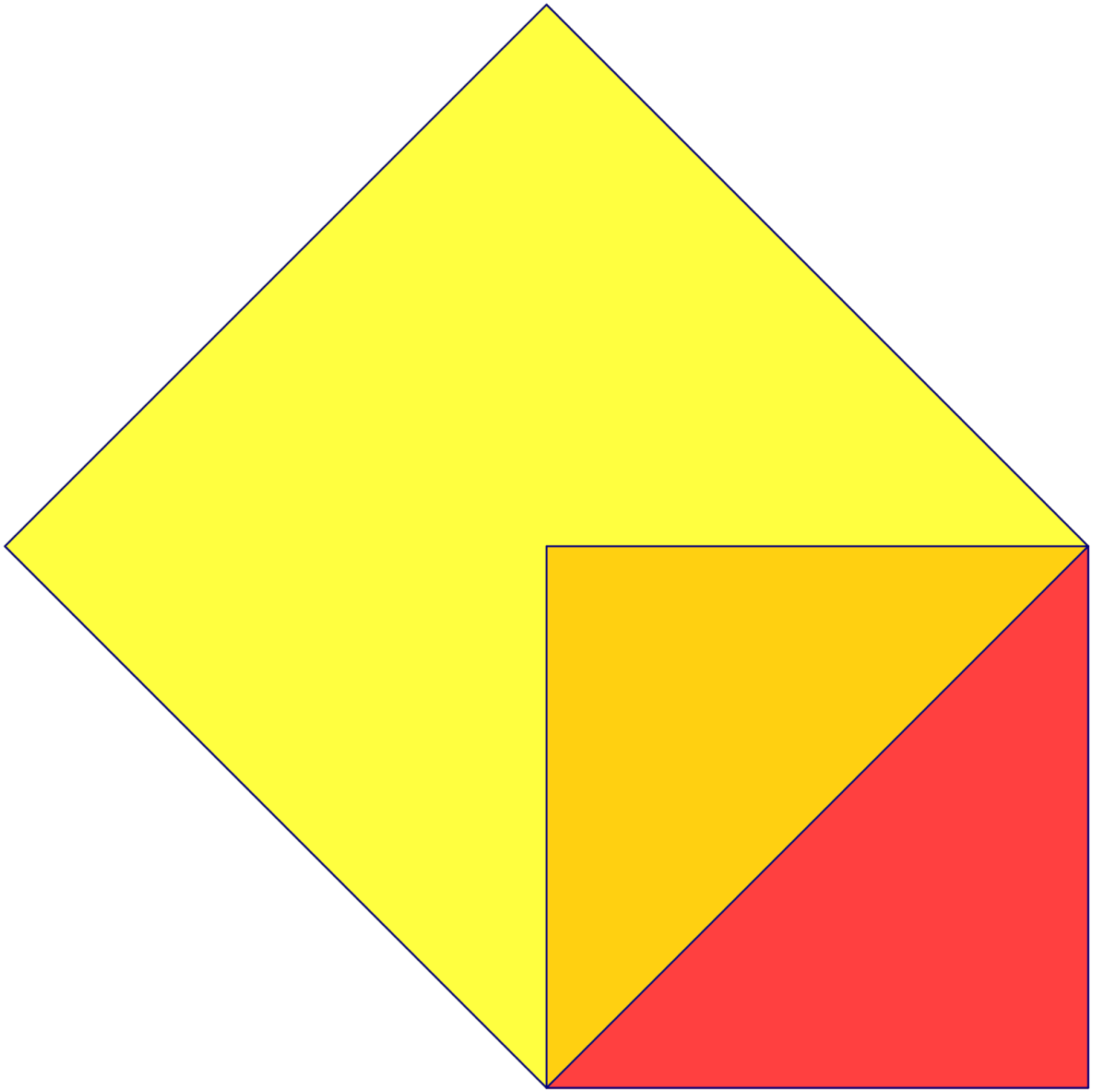

2

STARBUCKS
COFFEE
frappuccino
STARBUCKS
COFFEE
frappuccino



# Resources
## from 5280 Math

- Ten Plus One from the 5280 Math website
- How Open-Ended Math Tasks Keep on Giving
  from a middleweb.com post
- Noticing and Wondering: Pathways to Mathematical Meaning
  from an edcircuit.com post
- Advanced Common Core Math Explorations book series
- Problems That Never End and Projects That Never End

# Resources

- Mathematical Mindsets by Jo Boaler
- Extending the Challenge in Mathematics by Linda Jensen Sheffield
- Your ideas?

# Supporting each other's work

Share tasks, students' ideas, and/or classroom experiences.

jburkhart@5280math.com

http://www.5280math.com/ten-plus-one/