

10 Plus 1: Strategies for Math Depth and Complexity

Creating Deep Math Tasks 10 Strategies...

1. Write a story.
Create a story for a calculation or a real-world context for a math concept.
2. Draw a picture.
Draw a picture or diagram that shows the meaning of a concept or calculation.
3. Explain why.
Justify a prediction, claim, or answer using logic.
4. Find another way.
Find a different strategy or another answer.
5. Create examples.
Create multiple examples and non-examples of a concept.
6. Start with the answer.
Begin with the answer and find the "question."
7. Remove information.
Make a problem more open-ended by removing words or numbers.
8. Solve to learn.
Perform the task as a problem before a method has been taught.
9. Build a pattern.
Use the task to create or extend a pattern of numbers, equations, or shapes.
10. Change and observe.
Change one or more elements of the task, and observe the effects.

Creating Complex Math Tasks ...Plus 1

Use *more*.
digits, numbers, shapes, parts, steps, definitions, categories, relationships, etc.

Using the 10 Plus 1 Strategies

The Strategies:

- The *new tasks* are the goal.

The 10 Plus 1 Strategies are *tools* to spur your thinking to create new tasks. After you have used them for a while, you will learn to select strategies that are best for particular circumstances.
- You may use the 10 Plus 1 Strategies in any order.

You may also omit some of them. Some of the later strategies take more practice to use and tend to produce more challenging tasks.
- The 10 Plus 1 Strategies often work well in combination.

For example, Strategies 1, 2, and 3 usually make an excellent combination when the task is a computation.
- Different strategies may generate the same new task.

For example, Strategies 5 and 6 often lead to the same new task for certain concepts.
- A single strategy may generate more than one task.

This is especially true of strategies 9 and 10.
- You may use the tasks you create in many ways.

You may use them as the focus of a lesson, a pre- or post- assessment item, a small group activity, a homework assignment, a quick intervention for a student who has completed a classroom task and is waiting for something more challenging, etc. You will probably not use every task that you create.
- Depth and complexity interact.

In particular, increasing the complexity of a task often increases the depth. Offering varied levels of complexity may help you differentiate for many needs while maintaining focus on a central concept for large group discussion.
- You may use the 10 Plus 1 Strategies to build new tasks from other tasks *or* from concepts.

The examples:

- The examples are for your information. To use them with students, you may need to adapt the language or reading level to fit their age, developmental level, reading skills, knowledge of vocabulary, etc.

MATH DEPTH AND COMPLEXITY WORKSHEET

Original Task:

Underlying Concept:

Strategy _____ New Task:	Strategy _____ New Task:	Strategy _____ New Task:
Strategy _____ New Task:	Strategy _____ New Task:	Strategy _____ New Task:
Strategy _____ New Task:	Strategy _____ New Task:	Strategy _____ New Task:
Strategy _____ New Task:		Increasing Complexity Possible Modifications:

MATH DEPTH AND COMPLEXITY WORKSHEET

EXAMPLE FOR GRADES 1–2

Original Task: Round 67 to the nearest ten.

Underlying Concept: The meaning of rounding

Strategy <u> 1 </u> New Task: Describe a real-world situation in which you would round 67 to the nearest ten.	Strategy <u> 2 </u> New Task: Draw a diagram that shows why 67 rounded to the nearest ten is 70.	Strategy <u> 3 </u> New Task: Explain why 67 rounded to the nearest ten is 70.
Strategy <u> 4, 8 </u> New Task: Find a way (or a different way) to round 76 to the nearest ten.	Strategy <u> 5, 6 </u> New Task: A number rounded to the nearest ten is 70. What is the number? Or: Find six numbers between 60 and 70 that do <i>not</i> round to 70.	Strategy <u> 7 </u> New Task: Round 67 to the nearest <u> </u> ; Or, round <u> </u> to the nearest ten; Or, round <u> </u> to the nearest <u> </u> .
Strategy <u> 9, 6 </u> New Task: Find all whole numbers that equal 70 when rounded to the nearest ten. Do the same for 80, 90, 100, 110, etc. What patterns do you see?	Strategy <u> 10, 3 </u> New Task: Answer the original question. Then change it to "Round 67 to the nearest twenty." How does this affect the answer? Why?	Strategy <u> </u> New Task:
Strategy <u> </u> New Task:		Increasing Complexity Possible Modifications: Increase the size of the numbers students are rounding, or use decimals (if students know their meaning). Ask students to round to higher place values or decimal place values. Have students round to numbers that are not place values. Give students a large collection of data and ask them to sort the numbers into categories based on how they round.

Strategies for Math Depth and Complexity

Comments on the Example for Grades 1–2

Original Task: Round 67 to the nearest ten.

Underlying Concept: The meaning of rounding

Strategy 1

New Task: Describe a real-world situation in which you would round 67 to the nearest ten.

Strategy 2

New Task: Draw a diagram that shows why 67 rounded to the nearest ten is 70.

Comments: This might be a number line or a hundreds chart showing that 67 is closer to 70 than to 60. Or it could be a list of multiples of 10 with spaces between the numbers to show where 67 fits.

Strategy 3

New Task: Explain why 67 rounded to the nearest ten is 70.

Comments: Sometimes you can give students the answer, and ask them to explain why it is true. If students have already been taught a procedure, emphasize that they must explain *why*, not *how*. In other words, they use the *meaning* of rounding, not the procedure, to answer the question. In addition to developing reasoning, conceptual understanding, and math communication skills, this reduces the focus on right and wrong answers.

Strategies 4 and 8

New Task: Find a way (or a different way) to round 76 to the nearest ten.

Comments: Suppose you have not taught students a procedure for rounding. You have told them only that rounding to the nearest ten means finding the closest multiple of ten. Now students approach the task as a *problem* instead of a routine skill: They develop their own rounding strategies. For example: (1) Use a number line or a hundreds chart to find the closest multiple of ten visually; (2) Write out multiples of ten. Find the two closest numbers to 76 and calculate $70 - 67$ and $67 - 60$. Choose the multiple of ten with the smaller difference; (3) Use the ones digit as a guide to find the correct value of the tens digit (the traditional method).

Strategies 5 and 6

New Task: A number rounded to the nearest ten is 70. What is the number? Or: find at least seven numbers between 60 and 70 that do *not* round to 70.

Comments: Strategies 5 and 6 often work well together (and may be the same thing in some cases). Notice that the first question has more than one answer—even though the question does not say this. The second task encourages students to think about numbers between whole numbers.

Strategy 7

New Task: Round 67 to the nearest ____; Or, round ____ to the nearest ten; Or, round ____ to the nearest ____.

Comments: Allow students to choose the numbers and/or place values that they want to investigate. If they choose something too easy or too difficult, let them experiment before suggesting that they make a new choice. Make sure that the task remains a problem; i.e., don't tell them how to do it. But do ensure that they know what rounding *means*!

Strategy 9 (with 6)

New Task: Find all whole numbers that equal 70 when rounded to the nearest ten. Do the same for 80, 90, 100, 110, etc. What patterns do you see?

Strategy 10

New Task: Answer the original question. Then change it to "Round 67 to the nearest twenty." How does this affect the answer? Why?

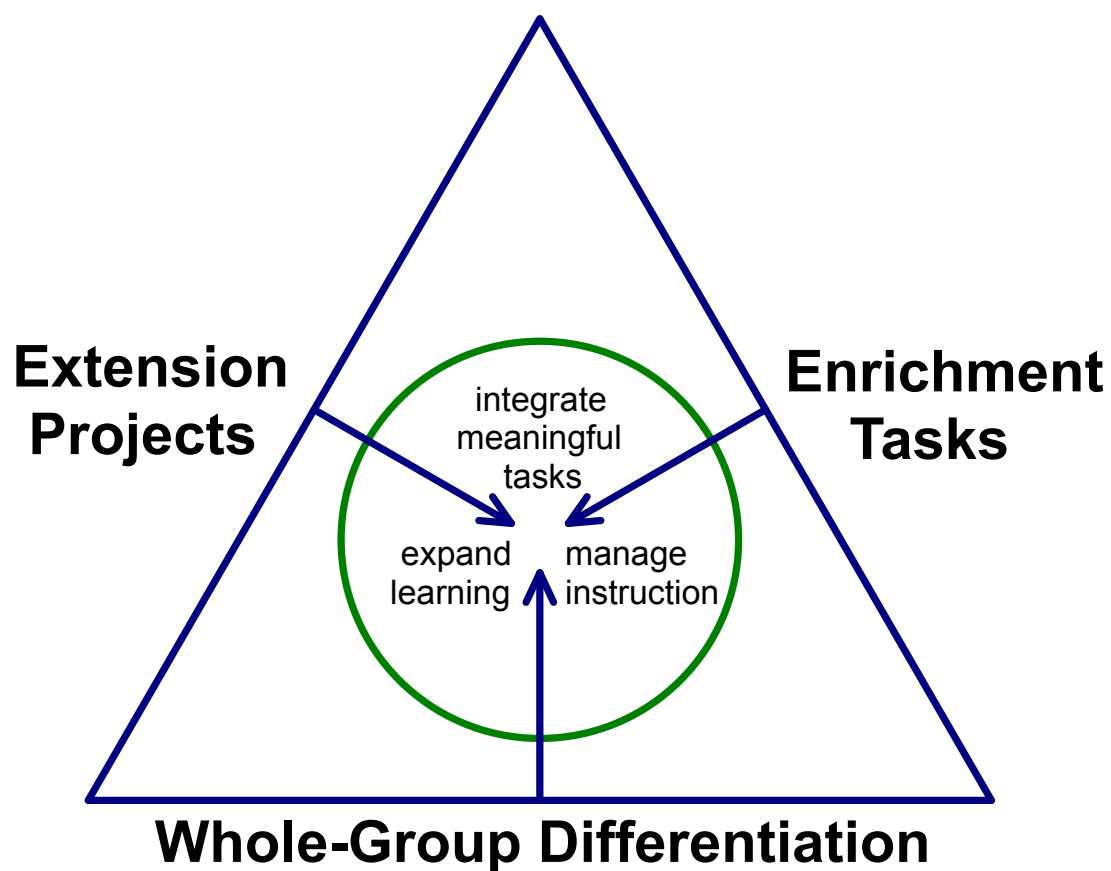
Comments: You could extend this task using Strategy 9: Find all whole numbers that equal 60 when round to the nearest twenty. Do the same for 80, 100, 120, etc. What patterns do you see?

Increasing Complexity

Modifications: Increase the size of the numbers students are rounding, or use decimals (if students know what they mean). Ask students to round to higher place values or decimal place values. Have students round to numbers that are not place values. Give students a large collection of data and ask them to sort the numbers into categories based on how they round.

Comments: Present these questions as problems. (i.e., Do not teach procedures for them.)

Components of Gifted Math Instruction



Resources for Advanced Math Students

Extension Projects and Enrichment Tasks

Extension Projects

- Long-term projects (one to two weeks)
- Aligned to math standards or curriculum
- Develop deep understanding of specific math concepts
- Focus on mathematical communication
- Highly challenging
- Require more facilitation by the teacher
- Require conversation and collaboration (student/student and teacher/student)
- Accessible to fewer students; expected of the most capable students
- Made available to many students
- Should be assessed

Enrichment Tasks

- Short-term tasks (or ongoing tasks that are easily "chunked" into small pieces)
- May be aligned to math standards or curriculum
- Develop general math problem solving and reasoning skills
- May require math communication
- Moderately challenging
- Designed to be done with little teacher support
- Done independently or in small groups
- Accessible to more students; good "go to" tasks to use when students are stuck on Extension Tasks
- Made available to all students
- May be assessed

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Education and experience

- B.A. Physics; M.A., Mathematics, M.A.T., Math Education
- 20 years teaching gifted math students
- 7 years professional development/coaching

Publications

Advanced Common Core Math Explorations: Factors and Multiples, 2014
Advanced Common Core Math Explorations: Numbers and Operations, 2104
Advanced Common Core Math Explorations: Fractions, 2104
Advanced Common Core Math Explorations: Measurement and Polygons, 2015

coming in 2016:

Advanced Common Core Math Explorations: Ratios, Proportions, and Similarity
Advanced Common Core Math Explorations: Probability and Statistics

all books published by Prufrock Press, www.prufrock.com

Building numbers from primes. *Mathematics Teaching in the Middle School*, NCTM, October, 2009.

Integer target: using a game to model integer addition and subtraction. *Mathematics Teaching in the Middle School*, NCTM, March, 2007.

Speaking, Professional Development, Consulting

Specialty: Building depth into mathematics instruction for gifted students

- Implementing *Advanced Common Core Math Explorations* in your school/district
- Implementing deep and challenging activities for elementary students
- Math instruction for gifted students: the big picture
- Using depth, breadth, and complexity to extend math standards
- Applying *10 Plus 1: Strategies to Increase Depth and Complexity of Math Tasks*
- Questioning techniques that support deep math learning
- Creating a classroom culture that supports mathematical challenge
- Motivating talented math students
- Assessment for talented math students

Presentations; interactive workshops; instructional modeling, collaboration, observation, and reflection; consulting

