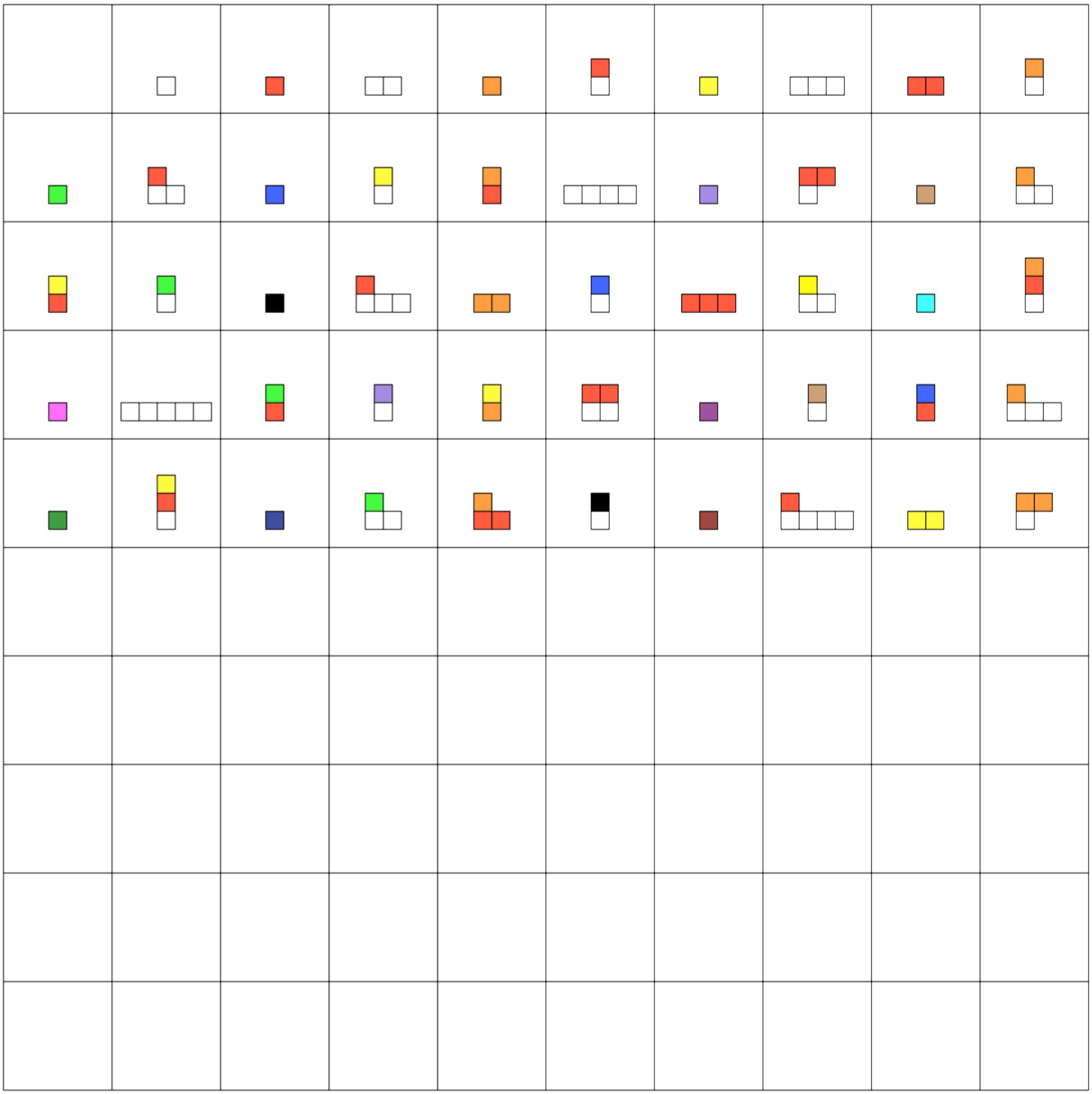
Building Blocks

1.



What is this about? What patterns do you see?
What will your students notice? What will they wonder?

based on selected concepts and activities from:
Advanced Common Core Math Explorations: Factors and Multiples
and *Advanced Common Core Math Explorations: Numbers and Operations*
by Jerry Burkhart, Prufrock Press, 2014
jerry@themathroom.org

2. What is special about the number 1?

3. If a number is divisible by 4, then…
   If a number is not divisible by 3, then…
   If a number is not divisible by 15, then…
   If a number is divisible by 3 and 5, then…
   If a number is divisible by 6 and 9, then…

4. What can you say about the block diagrams for 20 and 21?
   34 and 35? 63 and 64? 98 and 99? Why does this happen?

5. Can every natural number except 1 be "built"? Why or why not?

6. Is there ever more than one way to build a natural number?

7. What belongs in the 101st square on the grid? What about squares 199?
   200? 201? 999? 1000? 1001? 5292?
   Can you find efficient strategies to build the large numbers?

8. What do these calculations look like with the blocks? Do the blocks suggest any mental math strategies?

6 • 15	14 • 35	8 • 125	7 • 1	12 • 18
21 ÷ 3	98 ÷ 14	64 ÷ 32	5 ÷ 5	210 ÷ 42

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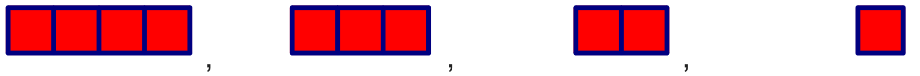
9. What concepts does this sequence illustrate?



10. What happens with the blocks when you do these calculations? What can students learn from this?

$8 \bullet 4$ $125 \bullet 5$ $64 \div 8$ $81 \div 27$

11. Show *all* factors of 84 with blocks and exponents. What do you observe?

12. Use blocks to find: GCF(65, 91); GCF(42, 70); GCF(26, 85).

13. $m - n = 6$. What can GCF(m, n) be? Why? How can the grid help you see this?

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14. What do the first five multiples of 12 look like with blocks? With exponents? What do you observe?

15. Use blocks to find: LCM(65, 91); LCM(48, 72); LCM(26, 85); LCM (1, 2, 3, 4, 5, 6, 7, 8, 9, 10).

What strategies might students use? What relationships might they discover? Which strategies still work for the last example?

16. How can you use blocks to simplify? $\frac{18}{30}$ $\frac{120}{210}$ $\frac{14}{98}$ $\frac{63}{64}$ $\frac{117}{250}$

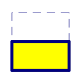
17. Use block diagrams to help predict the number of factors.
25, 16, 27 15, 21, 44 12, 18, 98 36, 100

Can you find a formula based on counting the colored blocks?

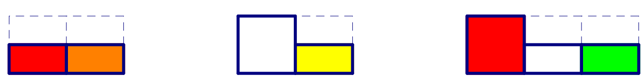
18. What do 6 and 6^2 look like with blocks and exponents? 14 and 14^2? 126 and 126^2? How can you recognize square numbers from their block diagrams (or exponential expressions)?

19. Use block diagrams (or exponents) to find the square roots: 49, 36, 64, 144, 196, 324, 10000.

20. What does this half-block represent? (Why is it *not* 3.5?) Write it in approximate form, then in an exact form two ways (with and without an exponent). Explain.



21. What do these block diagrams represent? How many different numerical expressions can you write for each? Explain.



22. How can you generalize these ideas to represent cube numbers, etc. with blocks?

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23. Understanding why $\sqrt{2}$ must be an irrational number.

- Suppose that $\sqrt{2} = \frac{a}{b}$ (*a* and *b* are natural numbers!)
- How can you write 2 in terms of *a* and *b*?
- Why do the numerator and the denominator each have an even number of blocks of each color?
- Why does the numerator have one more 2-block than the denominator?
- Why is this situation impossible? What does this mean about $\sqrt{2}$?
- How can you extend the argument to square roots of composite numbers?
- Where does the argument fail for the square root of a square number?

24. Predicting whether a fraction's decimal terminates or repeats.

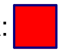
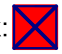
- Build the numerator and denominator.
- Simplify the fraction.
- What is always true of the blocks (prime factors) in the denominator of a terminating decimal? Why?
- If the decimal terminates, how can you use the denominator's block diagram to predict the number of decimal places? Explain.

25. Understanding why there are infinitely many prime numbers*.

- Supporting facts: (1) Every natural number except 1 can be "built" (has a prime factorization). (2) Successive counting numbers have no common colors (prime factors).
- Suppose there is a largest prime number, N. (indirect proof)
- Build the number T that contains one copy of every color (prime factor) from 2 to N: T = 2 • 3 • 5 • 7 • 11 • … • N.
- What can you say about the block diagram of T – 1 (or T + 1)?
- What does this tell you about trying to build T – 1? Why? What is the contradiction here?
- Why does this mean that the prime numbers cannot "end"?

*an informal way of looking at Euclid's famous proof

26. Suppose that each color of block has its own "anti-block." An anti-block is identified by an X.

red block:  red anti-block: 

When you join a block to its anti-block, both blocks disappear.

- What number does the red anti-block represent? Why?
- Write an equation that represents the joining of the two anti-blocks above.
- What is the real mathematical term for block/anti-block pairs?

27. What is the diagram for $\frac{1}{6}$? Why? Write an equation that shows the process of joining the two anti-blocks.

28. Represent $\frac{2}{3}$ by joining a block to an anti-block.

- What set of numbers is generated by all blocks and anti-blocks?
- What is the anti-diagram for $\frac{2}{3}$ (the diagram that makes it disappear)?
- How do you form the anti-diagram of a diagram?

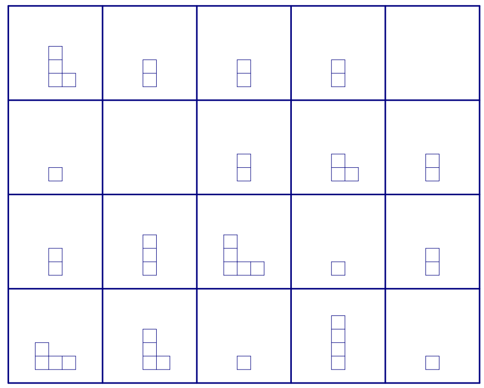
29. Show the following calculations with blocks and anti-blocks:

- $48 \bullet \frac{1}{6}$ and $48 \div 6$
- $5 \bullet \frac{1}{7}$ and $5 \div 7$ $\qquad$ $5 \div \frac{1}{3}$ and $5 \bullet 3$
  (Hint: Insert a block/anti-block pair to create a "convenient 1" for the division calculation. Compare this to the process of creating "convenient 0s" when using counters to represent integer subtraction.)
- $21 \bullet \frac{1}{12}$ and $21 \div 12$.

30. Show the following calculations with blocks and anti-blocks. (Students could do this before learning rules for multiplying and dividing fractions! In fact, they may discover some of the rules!)

$\frac{2}{7} \cdot \frac{3}{5}$ $\qquad$ $\frac{10}{21} \cdot \frac{14}{15}$ $\qquad$ $\frac{9}{10} \div \frac{3}{5}$ $\qquad$ $\frac{7}{13} \div \frac{3}{5}$ $\qquad$ $\frac{4}{21} \div \frac{18}{35}$

31. Find the values of the 20 consecutive natural numbers represented. Complete the two blank squares and color the blocks correctly. Create your own puzzles of this type!



32. If you allow blocks to represent variables (thus not limiting them to prime numbers), how can you use blocks to help students see the relationship between numeric factoring and algebraic factoring?

33. How can you use blocks to find GCFs and LCMs of pairs of algebraic expressions?

34. How can you use blocks to visualize and demonstrate these theorems? (All variables represent natural numbers.)

- If a is a factor of b, and b is a factor of c, then a is a factor of c.
- If a is a factor of b, and b is a factor of a, then $a = b$.
- If a is a factor of b, and c is a factor of d, then ac is a factor of bd.
- If a is not a factor of b, then na is not a factor of b.
- If na is a factor of b, then a is a factor of b. (What is the connection to the previous statement?)
- If a and b are factors of c, and if GCF(a, b) = 1, then ab is a factor of c. Why is the GCF condition needed?
- If a is a factor of bc, and GCF(a, b) = 1, then a is a factor of c.
- GCF(a, b) • LCM(a, b) = a • b. (See item 15.)
- GCF(a, b) is a factor of LCM(a, b).
- If GCF(a, b) = n, then GCF(a/n, b/n) = 1.
- If GCF(a, b) = 1, then GCF(a^n, b^n) = 1.

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Name____________________________ Activity _________________________________

Criterion	Description	Score
Depth of Understanding	• Know the *why* behind the *how*. • Understand the meanings of concepts. • Recognize and use connections between ideas.	
Problem Solving	• Create and use effective problem solving strategies. • Verify your results. • Solve the problem more than one way.	
Elaboration and Communication	• Give thorough, clear, concise explanations. • Use words, calculations, and diagrams effectively. • Support your explanations with examples.	
Generalizations and Reasoning	• Recognize, analyze, and extend patterns. • Make and test predictions. • Use logic to evaluate claims and justify conclusions.	
Correctness and Precision	• Give correct answers stated with appropriate precision. • Calculate accurately and efficiently. • Use mathematical vocabulary correctly and precisely.	
Originality and Extensions	• Invent ideas and strategies that were not taught. • Find ideas and strategies that are rarely discovered. • Propose new ideas or questions to study.	
Effort and Perseverance	• Show consistent effort. • Make progress appropriate to your understanding. • Persist through difficulties.	

A suggested scoring system:

5	evidence of learning beyond the level of course standards
4	evidence of learning at the level of course standards
3	evidence of learning approaching the level of course standards
2	evidence of learning below the level of course standards
1	evidence of learning significantly below the level of course standards
0	little or no evidence of progress toward meeting course standards

No numerical scoring system will ever replace the value of a few thoughtful written comments related to students' ideas! Of course, you may also incorporate other criteria such as legibility, organization, mechanics (spelling, punctuation, and grammar), etc. However, your scoring system should reflect the central goal of mathematical learning.

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Publications

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Advanced Common Core Math Explorations: Numbers and Operations, 2104
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coming in 2016:
Advanced Common Core Math Explorations: Ratios, Proportions, and Similarity
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Building numbers from primes. *Mathematics Teaching in the Middle School*, NCTM, October, 2009.
Integer target: using a game to model integer addition and subtraction. *Mathematics Teaching in the Middle School*, NCTM, March, 2007.

Education and experience

- B.A. Physics; M.A., Mathematics, M.A.T., Math Education
- 20 years teaching gifted math students
- 7 years professional development/coaching