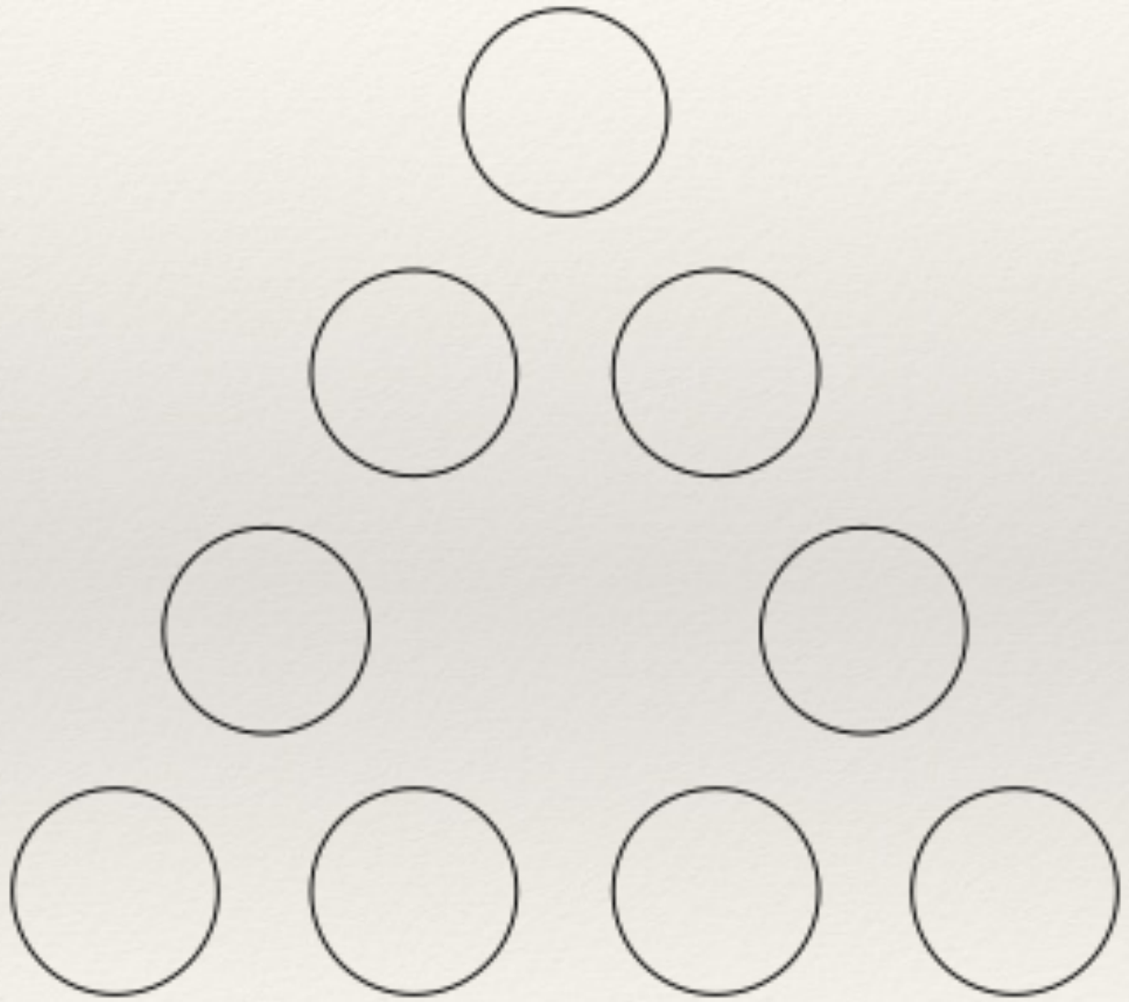
I notice… *I wonder…*



from *Advanced Common Core Math Explorations: Numbers and Operations*
by Jerry Burkhart, Prufrock Press: 2014

CAGT, 2015

Noticing and Wondering Your Way to Mathematical Challenge

Presenter: Jerry Burkhart

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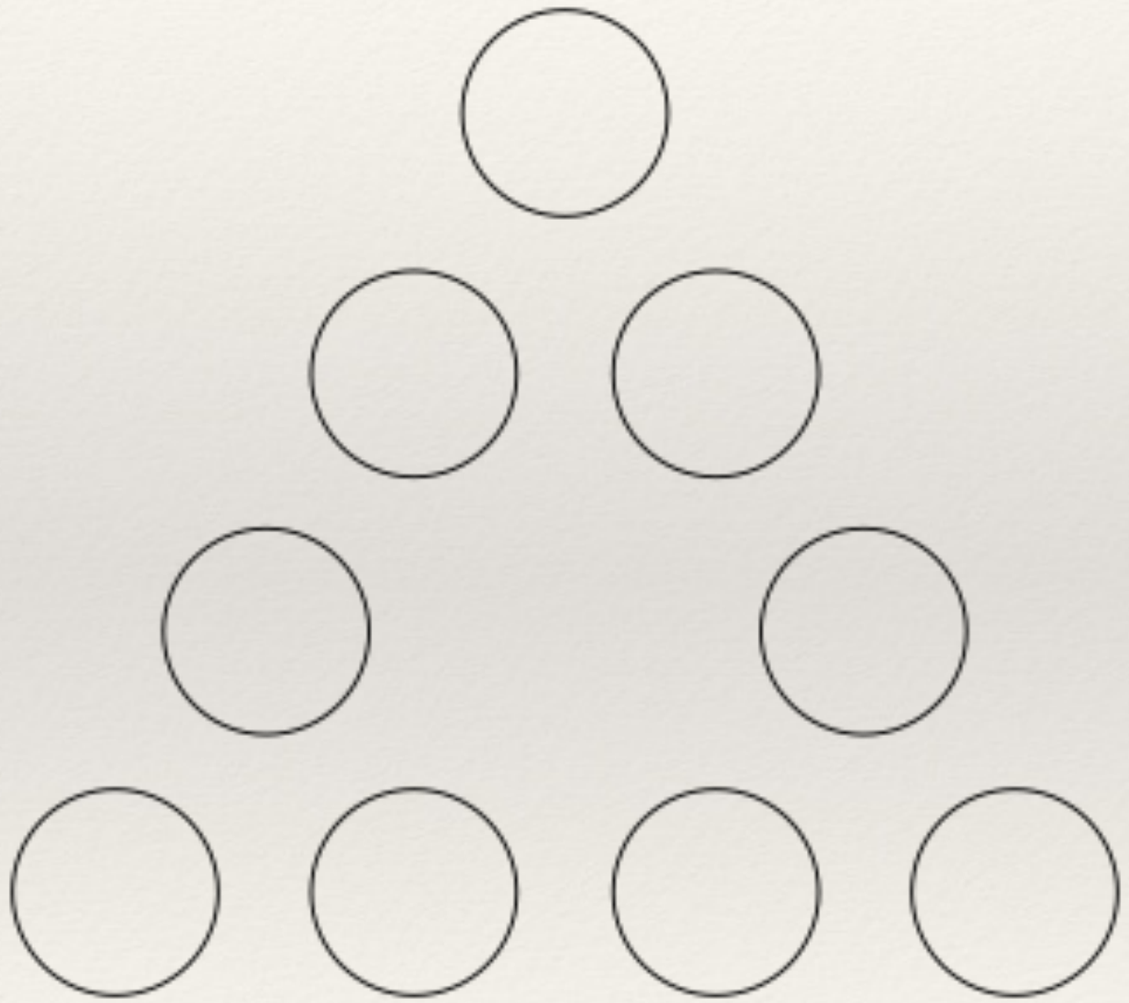
Goals

- Use *noticing* and *wondering* as tools to spur analytical and creative thinking in math.
- Provide meaningful alternatives to "teaching the next topic" when differentiating for advanced math learners.
- Examine the meaning of depth and complexity in the context of math.
- Explore examples of deep and complex math tasks.

I notice… *I wonder…*

1, 2, 3, 4, 5, 6, 7, 8, 9



from *Advanced Common Core Math Explorations: Numbers and Operations*
by Jerry Burkhart, Prufrock Press: 2014

Noticing

Helpful for clarifying, analyzing, summarizing

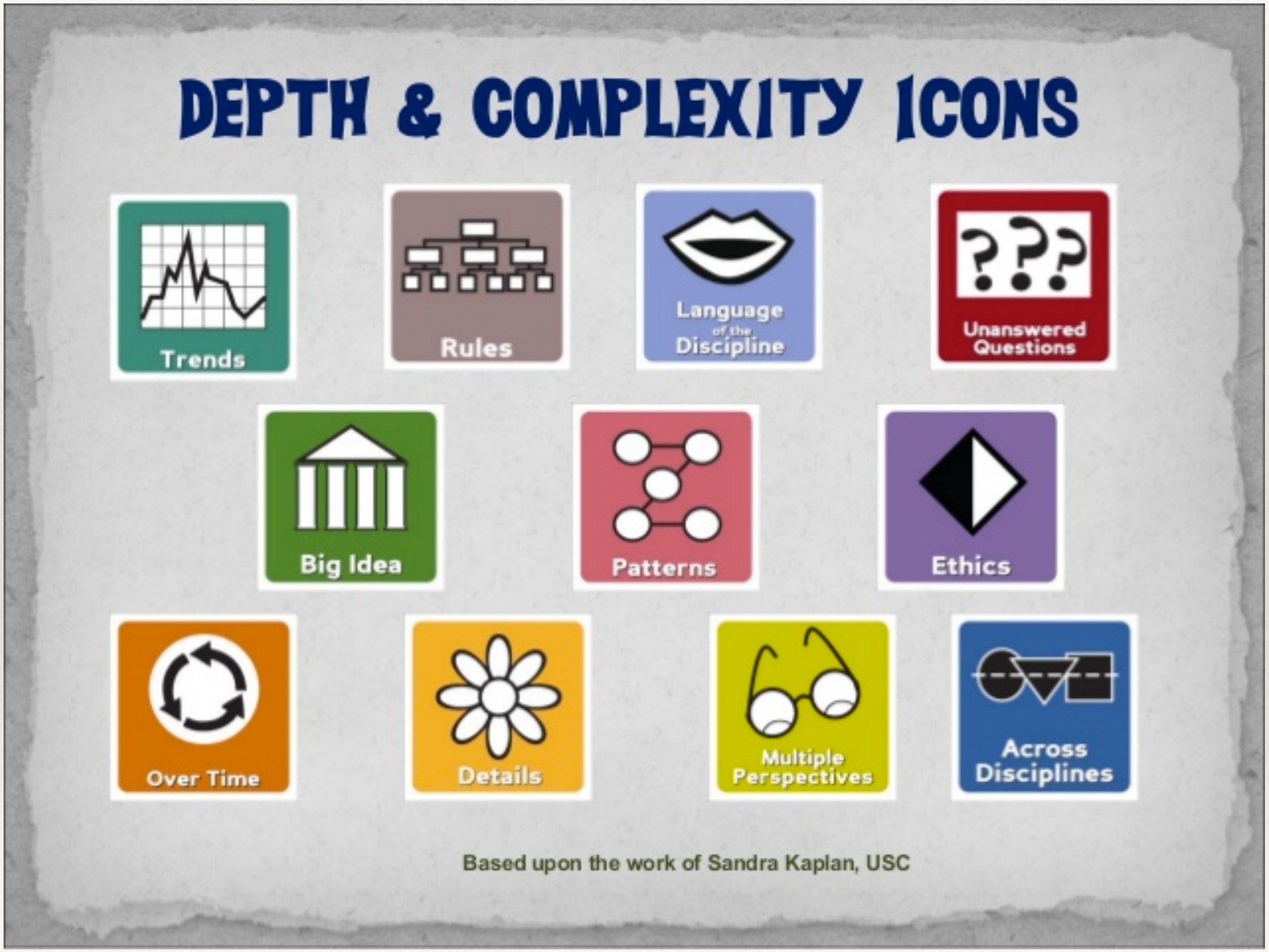
- Facts
- Estimates
- Characteristics or Properties of numbers or shapes
- Similarities and Differences
- Patterns
- Relationships and Connections
- The effects of change

Wondering

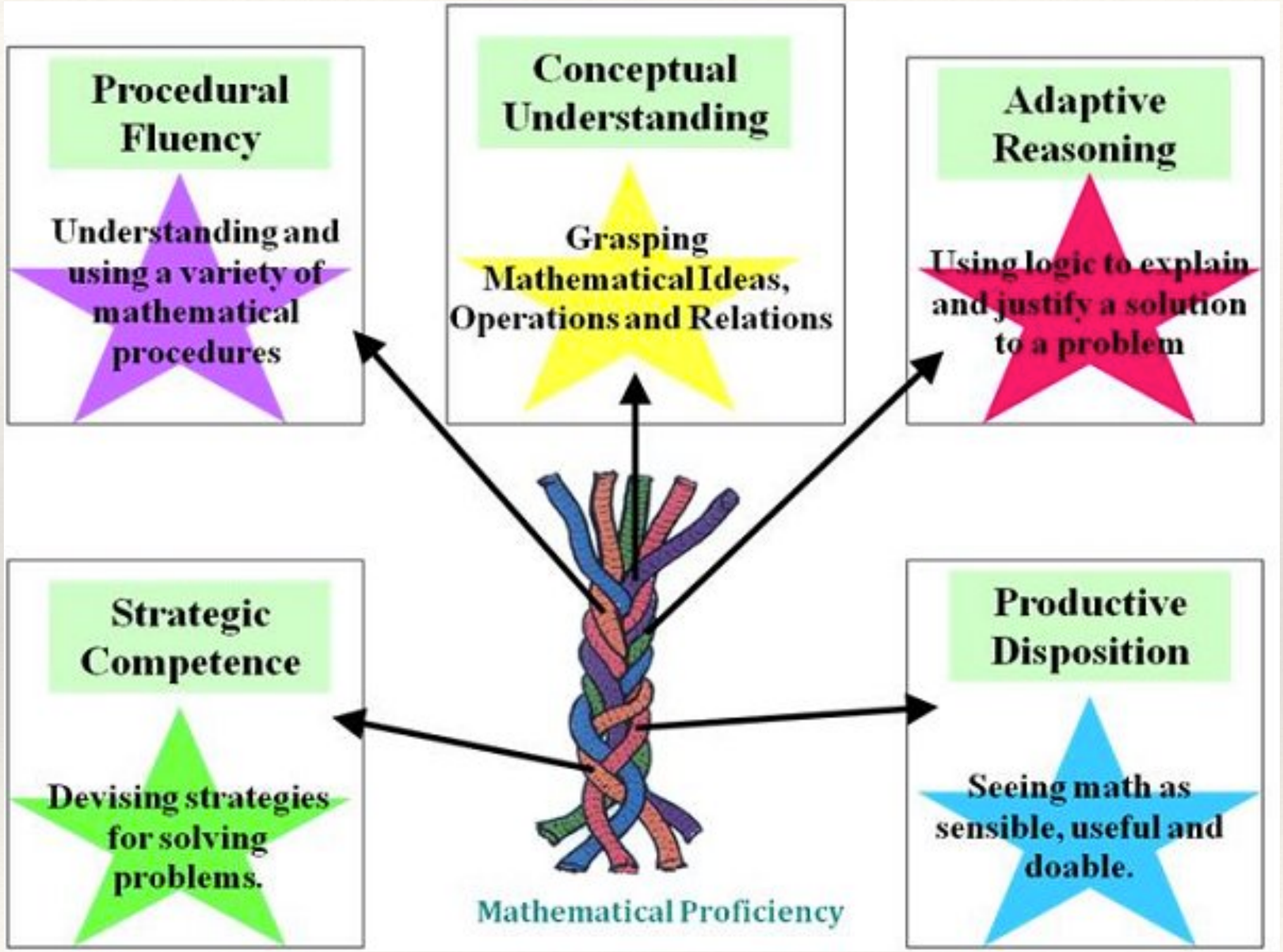
Helpful for creating, predicting, reasoning

- What is this about?
- Is there another solution?
- How many solutions are there?
- Are there patterns in the solutions?
- How do know when I've found all solutions?
- What comes next?
- Why is this true? Why does this work?
- What can be changed?

Kaplan's Icons



Math Proficiency Strands



Kilpatrick, J., Swafford, J., Findell, B. (Ed.). (2001). Adding it up: helping children learn mathematics. Washington, DC: National Academy Press.

NCTM Process Standards

- Problem Solving
  - Develop, apply, and verify your own strategies to answer questions.
- Reasoning and Proof
  - Make and test predictions. Analyze and extend patterns. Justify conclusions.
- Communication
  - Organize, record, and present mathematical ideas clearly (orally and in writing).
- Connections
  - Recognize relationships among mathematical ideas and between math and other disciplines.
- Representations
  - Model math concepts with words, graphs, tables, symbols, pictures, manipulatives, etc.

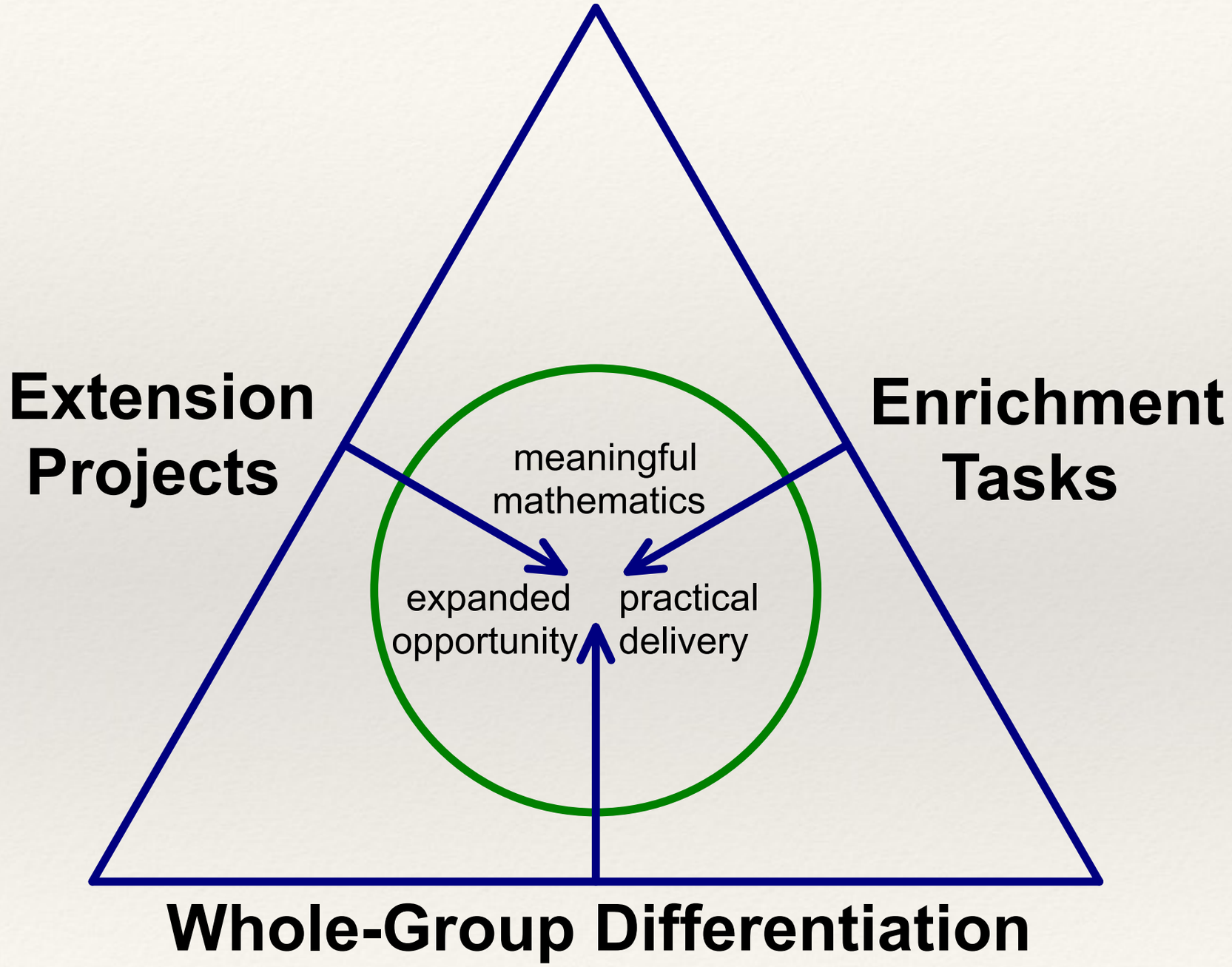
adapted from *Principles and Standards for School Mathematics*. Reston, Va.: NCTM, 2000.

Connecting Kaplan to Best Practices in Math

Conceptual Understanding	**Big Idea**, Patterns, Trends, Different Perspectives
Procedural Fluency	**Rules**, Ethics, Different Perspectives
Adaptive Reasoning	**Patterns** and **Details**, Trends, Different Perspectives
Strategic Competence	**Unanswered Questions**, Different Perspectives, Ethics
Mathematical Communication	**Language of the Discipline**, Rules, Different Perspectives
Connections	**Across the Disciplines**, Different Perspectives, Patterns
Representations	**Different Perspectives**, Language of the Discipline

Math Instruction Triangle

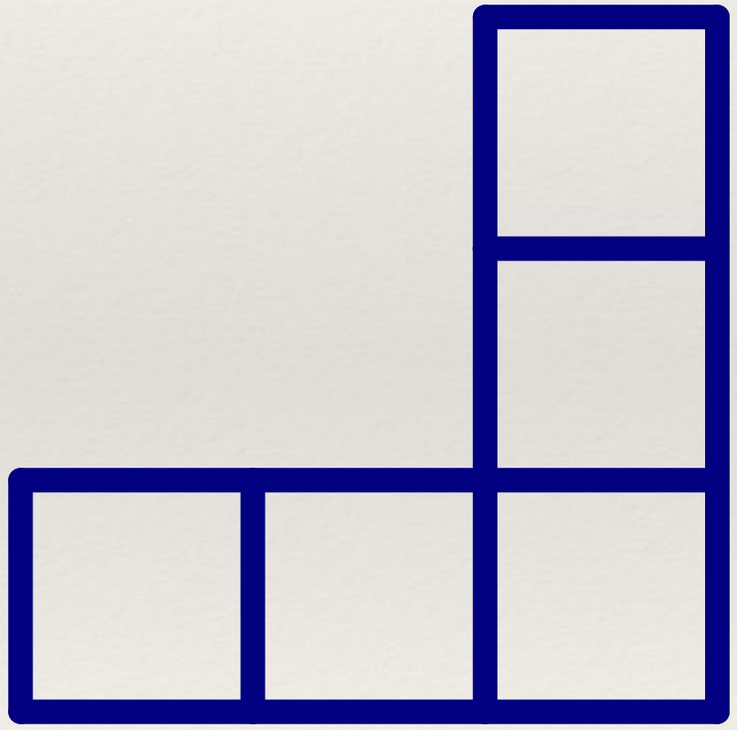
for advanced learners



Jerry Burkhart, 2015

I notice… *I wonder…*

1, 2, 3, 4, 5



from *Extending the Challenge in Mathematics: Developing Mathematical Promise in K-8 Students*
by Linda Jensen Sheffield, Corwin Press: 2003

I notice… *I wonder…*

$$0 = 4 - 4$$

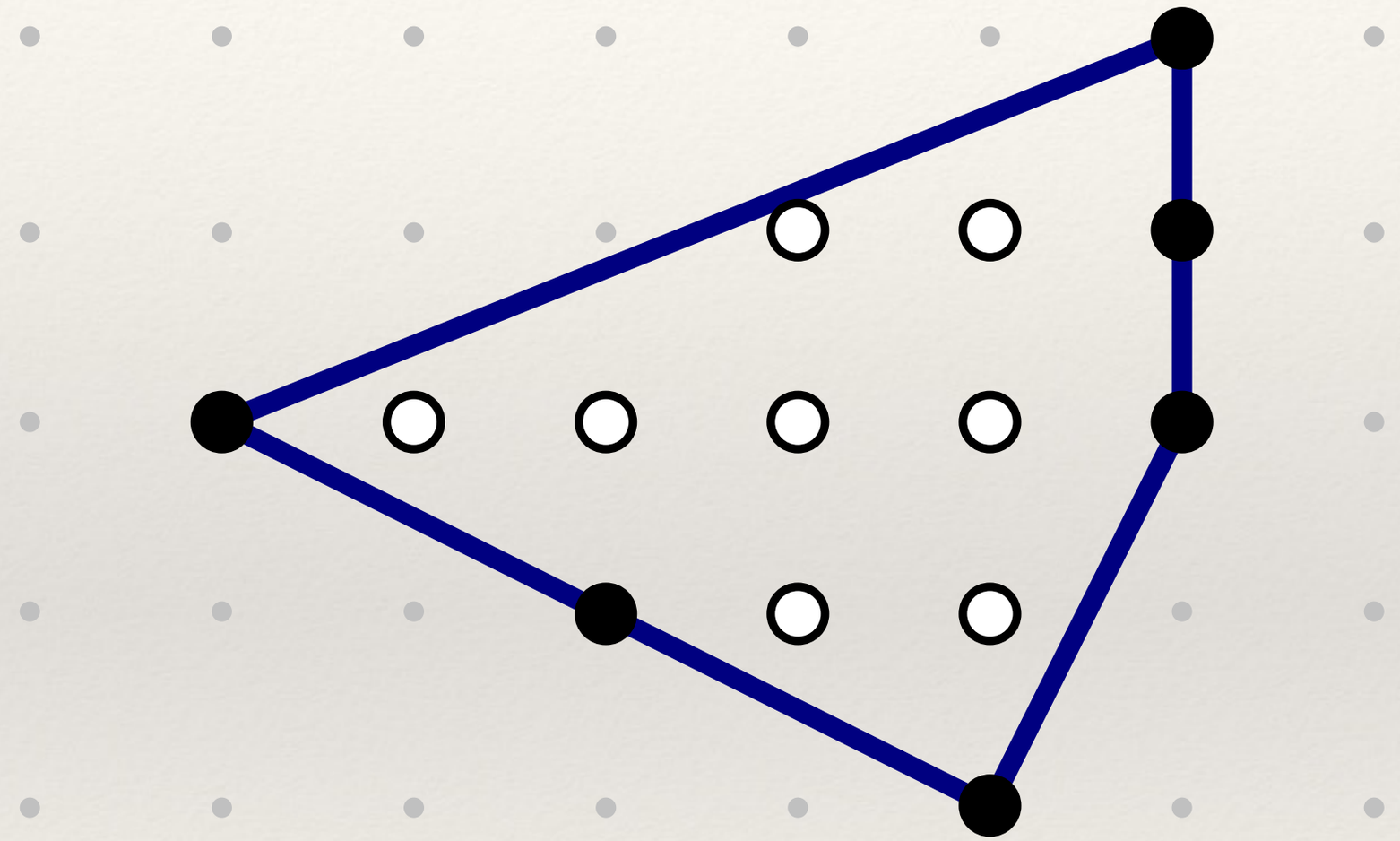
$$1 = 4 - 3$$

$$2 = 4 - 2$$

$$3 = 4 - 1$$

I notice… *I wonder…*

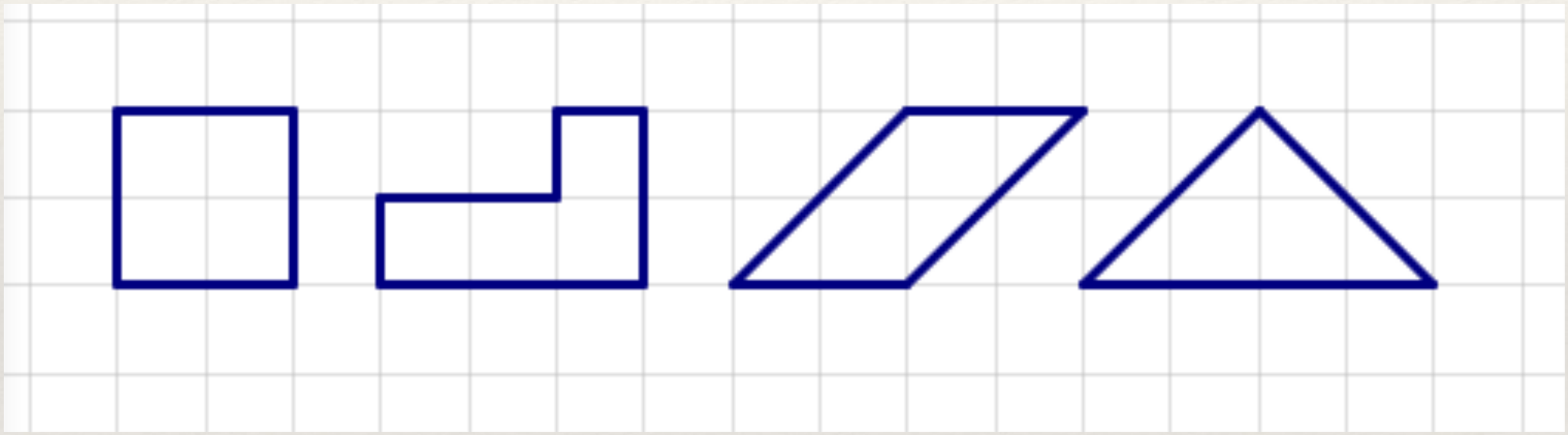
I notice… *I wonder…*



I notice… *I wonder…*

x 4 ÷ 8

I notice… *I wonder…*



I notice… *I wonder…*

·	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Jerry Burkhart, 2015

I notice… *I wonder…*

8	1	6
3	5	7
4	9	2

I notice… *I wonder…*

8	1	6
3	5	7
4	9	2

3	3	3	3	
				3
				3
				3
				3

I notice… *I wonder…*

Our Numeral	Torran Numeral
4	10
8	20
11	23
15	33
19	103

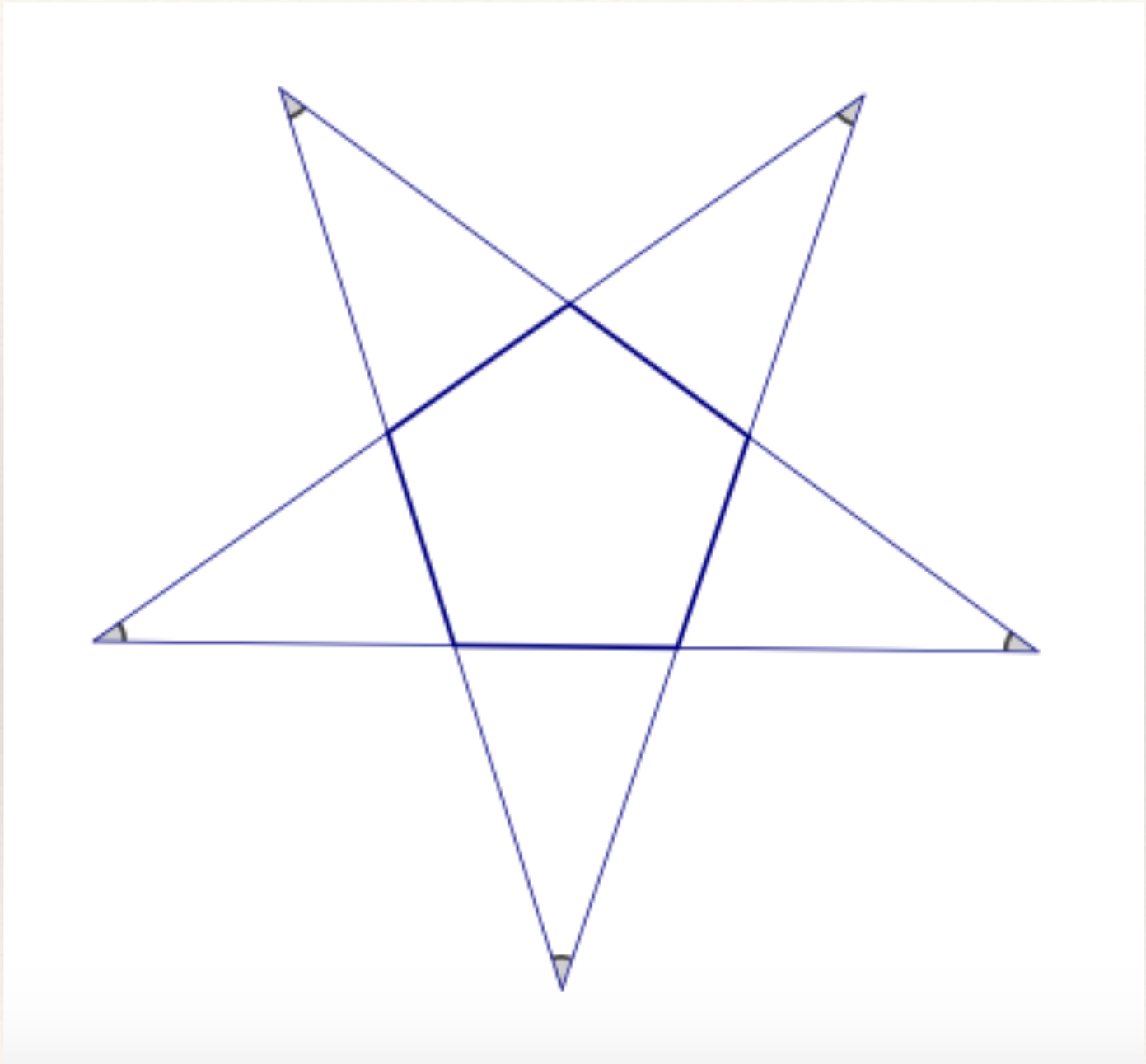
from *Advanced Common Core Math Explorations: Numbers and Operations*
by Jerry Burkhart, Prufrock Press: 2014

I notice… *I wonder…*

$$\frac{3}{5} \quad \frac{11}{18} \quad \frac{8}{13} \quad \frac{13}{21} \quad \frac{5}{8}$$

from *Advanced Common Core Math Explorations: Ratios, Proportions, and Similarity*
by Jerry Burkhart, Prufrock Press: 2015

I notice… *I wonder…*



from *Advanced Common Core Math Explorations: Measurement and Polygons*
by Jerry Burkhart, Prufrock Press: 2014

I notice…	*I wonder…*
$2+2$	$2\cdot 2$
$-1+\frac{1}{2}$	$-1\cdot\frac{1}{2}$

from *Advanced Common Core Math Explorations: Fractions*
by Jerry Burkhart, Prufrock Press: 2014

I notice… *I wonder…*

Bristlecone School District data

Mountain Heights Middle School:

51,600 square feet of floor space

470 students

North Star Middle School:

118,300 square feet of floor space

725 students

from *Advanced Common Core Math Explorations: Ratios, Proportions, and Similarity*
by Jerry Burkhart, Prufrock Press: 2015

I notice… *I wonder…*

Number	Code	Number	Code
0	none	8	3
1	0	9	20
2	1	10	101
3	10	11	10000
4	2	12	12
5	100	13	100000
6	11	14	1001
7	1000	15	110

from *Advanced Common Core Math Explorations: Factors and Multiples*
by Jerry Burkhart, Prufrock Press: 2014

I notice… *I wonder…*

$$4^4 = 256$$

$$4^3 = 64$$

$$4^2 = 16$$

$$4^1 = 4$$

I notice… *I wonder…*

$$4^2 = 16$$

$$4^1 = 4$$

$$4^0 = 1$$

$$4^{-1} = \frac{1}{4}$$

I notice…	*I wonder…*
$4^2 = 16$	$9^2 = 81$
$4^1 = 4$	$9^1 = 9$
$4^0 = 1$	$9^0 = 1$
$4^{-1} = \frac{1}{4}$	$9^{-1} = \frac{1}{9}$

10 Strategies to Increase Depth of Math Tasks

1. Write a Story.
2. Draw a Picture.
3. Find another way.
4. Explain why.
5. Create examples.
6. Start with the answer.
7. Remove information.
8. Solve to learn.
9. Build a pattern.
10. Ask “What if?”

Jerry Burkhart, October 2015

Strategies to Increase Complexity of Math Tasks

Use more…

- digits.
- numbers.
- procedures.
- parts.
- categories.
- relationships.

Resources

- *Principles and Standards for School Mathematics*. Reston, Va.: NCTM, 2000.
- *Helping Children Learn Mathematics*, by the National Research Council, 2000.
- *Extending the Challenge in Mathematics*, by Linda Jensen Sheffield. Corwin Press, 2003.
- *Advanced Common Core Math Explorations* series, by Jerry Burkhart. Prufrock Press, 2014 – 2015
- *Good Questions: Great Ways to Differentiate Math Instruction*, by Marian Small. Teacher's College Press, 2012.

Thank you!

Jerry Burkhart

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Advanced Common Core Math Explorations books:

http://www.prufrock.com