Ten Plus One

Ten strategies...
for creating deeper math tasks

1. Write a story.
   Create a story for a calculation or a real-world context for a concept.

2. Draw a picture.
   Draw a picture or diagram that shows the meaning of a concept or calculation.

3. Explain why.
   Justify a prediction, claim, or answer using logic.

4. Find another way.
   Find a different strategy or another answer.

5. Compare and contrast.
   Compare and contrast expressions, shapes, patterns, strategies, representations, etc.

6. Start with the answer.
   Begin with the answer and find the "question."

7. Remove information.
   Remove words, numbers, or other elements of the task.

8. Solve to learn.
   Perform the task as a problem before learning a method.

9. Build a pattern.
   Make or extend a pattern of numbers, equations, or shapes based on the task.

10. Ask "What if...?"
    Change one or more elements of the task. Observe the effects.

...**Plus One** strategy
for creating more complex math tasks

Use *more...*

digits, numbers, shapes, parts, variety, steps, ideas,
information, definitions, categories, or relationships.

Ten Plus One
FAQ

What is Ten Plus One?

Ten Plus One is a tool for modifying basic math tasks in order to make them deeper and/or more complex. Though it was designed to help teachers meet needs of talented students, you may use it to differentiate tasks for all learners.

How will my students benefit from Ten Plus One?

The tasks that you create with Ten Plus One can help your students learn to think more independently, clearly, and creatively about mathematical concepts and problems.

Why are depth and complexity so important?

Depth and complexity are two key features of tasks that meet talented students' needs. Deep mathematical learning enables all students to *retain* what they learn, *transfer* it to support new learning, *apply* it in novel situations, and *appreciate* the value of math.

Adjusting the complexity of a math task allows you to differentiate it for learners with different backgrounds or needs. Math tasks with greater complexity can develop students' executive function skills and increase opportunities to discover new ideas and make new connections.

What do deep math tasks look like?

Deep math tasks involve the *why* of math concepts, not just the how. They explore the *meanings* of the concepts and make *connections* between ideas. Depth also comes from a learner's approach to a task. Students can learn to think more deeply about almost any math task by asking questions differently.

Less Depth	**More Depth**
What do I *do*?	What do I *think*?
What are the *steps*?	What does it *mean*?
How can I *remember*?	How does it *connect* to what I know?

What do complex math tasks look like?

Complex tasks tend to be "messy," because students are asked to manage a lot of information. However, the tasks should not be busy work. In other words, students should not simply be doing longer versions of the original task. The best complex tasks naturally have more depth as well, because they contain new ideas, and they require students to extend concepts and explore new relationships.

How do I use the Ten Plus One strategies?

Ten Plus One is easy to use. Just follow this five-step process!

1. *Identify* a math task.
2. *Choose* a depth strategy.
3. *Apply* the strategy to enhance the task.
4. *Adjust* the complexity (optional).
5. *Anticipate* students' thinking.

See the following pages for a template and examples.

Are some strategies easier to use than others?

Most teachers find that the earlier strategies, especially numbers 1–4 are the easiest to use at first. Numbers 9 and 10 especially may take more practice.

As you gain more experience with Ten Plus One, you will discover that certain strategies seem to work best with certain kinds of tasks. However, many tasks can be enhanced in different ways with all ten strategies!

Can I combine strategies?

Yes! Some of the best new tasks come from mixing and matching strategies. You may discover that certain combinations of strategies work especially well together. For example, when the task is a calculation, strategies 1, 2, and 3, often make a great combination!

Note that different strategies may sometimes lead to the same task.

Where does the complexity strategy fit in?

You may use the complexity strategy by itself if you like, especially if you are creating multiple versions of the same task to meet diverse needs. If you are creating a new task for advanced learners, you will usually enhance the depth before the complexity. See the examples on the following pages to get a feel for how this looks.

What kinds of tasks can I create?

You can use Ten Plus One to create enhanced tasks for all sorts of situations: "bell-ringers" to jump-start your students' thinking at the beginning of a lesson, individual or group tasks during a classroom lesson, formative or summative assessment items such as exit slips or end-of-unit test questions, take-home tasks, etc. Once you have gained some experience, you may even use them to create tasks "on the fly" for students who finish their work early.

Can I use Ten Plus One to help differentiate my whole-group lessons?

Yes! Use the complexity strategy to create two or three versions of the same task, each with a different level of complexity. The least complex version should be accessible to all students, while the most complex version should create learning opportunities for the strongest students. Students work independently or in small groups on the version(s) of their choice. Afterwards, the whole class discusses their strategies and answers. The discussion will be meaningful to all students, because all of them are working on essentially the same problem but at their own level of thinking.

What if the strategies do not suggest any ideas for a new task?

(1) Look at some of the examples in the following pages to see if they spark any new ideas. (2) Build a new task based on the *objectives* of the task instead of the task itself. (3) Collaborate with a colleague. Note: As you gain more and more experience, you will find it easier to think of new tasks.

How can I tell if I am using the Ten Plus One strategies correctly?

The Ten Plus One strategies are a tool. There are no strict rules for using them. If they help you to create meaningful tasks that extend your students' mathematical thinking, then you are using them correctly.

What are the best ways to teach with the new tasks that I create?

Ten Plus One helps you to create tasks that elicit *students'* thinking. Take advantage of this wonderful opportunity!

1. Step back. Let your students do most of the thinking.
2. Allow your students to collaborate.
3. Create discussions in which your students share, compare, and justify their thinking strategies.
4. Focus more on process than on answers.
5. Expect students to communicate their ideas for some problems in writing.
6. After a lesson (or after collecting students' work), make note of important ideas, strategies, misconceptions, etc. that you heard or saw. Use these to adjust the task and/or guide your planning for the next time you use the task.
7. Save some of your students' work as exemplars and for discussion with colleagues.

Ten Plus One Template 1

1. Identify a math task.	5. Anticipate students' thinking.
2. Choose a depth strategy.	
3. Apply the strategy.	
4. Modify the complexity.	

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Ten Plus One Template 2

1. Identify a math task.	5. Anticipate students' thinking.
2. Choose a depth strategy.	
3. Apply the strategy.	
4. Modify the complexity.	

1. Identify a math task. 35 – 18	5. Anticipate students' thinking. For students who are already proficient at creating subtraction stories and using place value to justify their calculations, it may be necessary to modify the complexity right away. Rather than making the calculation messier by making the numbers larger, the complex tasks increase the number of items being subtracted or added (which will make students think harder about their stories) and require students to think about more than one meaning for subtraction.
2. Choose a depth strategy. 1. Write a story. 2. Draw a picture. 3. Explain why.	
3. Apply the strategy. Write a story problem for 35 – 18. Draw a picture to show the problem and the answer. Use the picture to explain why the answer makes sense.	
4. Modify the complexity. Change 35 – 18 to 35 – 18 + 13 and/or write more than one story; for example, one using "take away" and another using "how many more."	

1. Identify a math task. $6 \div \frac{2}{3}$	5. Anticipate students' thinking. Possible misconceptions: • Dividing by 2/3 makes the answer smaller than 6 (possible error: 4). • Dividing by 2/3 makes the answer 1 2/3 times greater (possible error: 10). Possible correct understandings: • Count the number of groups of 2/3 in 6. • Find the number that 6 is 2/3 of. • Depending on their diagrams/meanings, students may use 6 ÷ 2 • 3 or 6 • 3 ÷ 2. Students may use number lines, rectangles, circle diagrams, arrays, pictures of objects, etc. Students who finish early may change the dividend from 6 to 5. Even though 5 is less than 6, this makes the problem *more* complex, because the answer is not a whole number.
2. Choose a depth strategy. 1. Write a story. 2. Draw a picture. 3. Explain why.	
3. Apply the strategy. Write a story problem for $6 \div \frac{2}{3}$. Draw a picture to show the problem and the answer. Use the picture to explain why the answer makes sense.	
4. Modify the complexity. Write a story problem for $5 \div \frac{2}{3}$. Draw a picture to show the problem and the answer. Use the picture to explain why the answer makes sense.	

1. Identify a math task.

100 x 37

2. Choose a depth strategy.

4. Find another way.
3. Explain why.

3. Apply the strategy.

Find another way to multiply 10 by 37. Explain why your method makes sense.

4. Modify the complexity.

Find three ways instead of two. Give reasons for each method. And/or change the numbers: Multiply 100 by 370 or 100 by 3.7, etc.

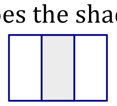
5. Anticipate students' thinking.

Note: Use this task before discussing shortcuts for multiplying numbers by 100 (or even 10).

- Some students may already know the shortcut, but they should be able to explain why it works and to think of more methods.
- Students may think of 100 groups of 37 or 37 groups of 100. Discuss advantages and disadvantages of each.
- Students may use base 10 blocks (actual or pictures).
- Students may imagine arrays (without drawing all of the dots) and show groups.
- Students may think of 10 groups of 100, 20 groups of 100, 30 groups 100, and then add 7 groups of 100.
- Some students may think of 100 as 10 groups of 10.

1. Identify a math task.

What fraction does the shaded part show?

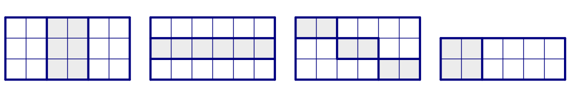


2. Choose a depth strategy.

5. Compare and contrast.

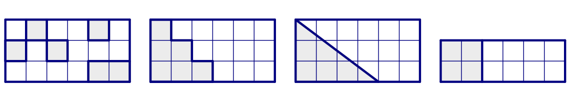
3. Apply the strategy.

What do the first three pictures have in common? What do they all have in common?



4. Modify the complexity.

Use the same questions with these pictures.



5. Anticipate students' thinking.

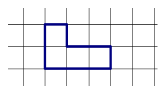
Some students may count squares and notice that the first three pictures have six shaded squares, but the last has only four.

In order to find something that the last picture has in common with the first three, students may need to compare the shaded part to the entire rectangle and notice that three copies of the shaded part fit into the whole thing.

This may be harder for them to see in the third picture, because the pieces have to be rearranged.

The complex pictures require more imagination. In the first picture, the parts are not all connected. The third picture is more challenging because it includes parts of squares that are not as easily rearranged. Some students may notice that two copies of the triangle make a rectangle with twelve squares.

1. Identify a math task.
Find the area.



2. Choose a depth strategy.
6. Start with the answer.
5. Compare and contrast.

3. Apply the strategy.
The area is four square units. Draw at least five polygons. How else are the shapes the same? How are they different?

4. Modify the complexity.
Ideas: Increase the number of drawings. Ask students to draw specific kinds of shapes such triangles. Encourage them to use quarter-squares in some drawings. Make the area a mixed number like 6½.

5. Anticipate students' thinking.

Students may begin by creating simple shapes out of whole squares. After they've created a few, the discussion may branch off into new directions with questions like:

- Is this shape a polygon?
- Does my shape have to be all in one piece?
- Is it okay to use parts of squares?
- Can I try to make my polygons look like a picture of something?
- Do my shapes all have the same perimeter?
- Can I use length x width to find the areas of my shapes?
- Are there formulas for these shapes?

1. Identify a math task.

36 ÷ 9 = ____

2. Choose a depth strategy.
7. Remove information.
9. Build a pattern.

3. Apply the strategy.

36 ÷ ____ = ____
Look for patterns in your answers and use them to predict more solutions.

4. Modify the complexity.
Ideas: (1) Change 36 a larger number so that students need to search for factors that they have not memorized. (2) Add a blank.
36 ÷ ____ ÷ ____ = ____
(or let the last blank be 4).

5. Anticipate students' thinking.

After students have exhausted the familiar possibilities, they may be surprised to discover that there is still a lot to learn.

Patterns in the list:
1, 36 2, 18 3, 12 4,9 6, 6
36, 1 18, 2 12, 3 9, 4 6, 6

- All pairs have a product of 36.
- When you find one pair, you can reverse it.
- When the dividend gets larger, the divisor gets smaller.
- When the dividend multiplies by 2 (or *x*), the divisor divides by 2 (or *x*).

Students may use these patterns to explore other possibilities. What happens when the divisor is ½ or some other number less than 1? Does it make sense for the divisor to be 0? What happens if the dividend is greater than 36? What happens if you divide by a mixed number like 1½?

1. Identify a math task. $\frac{1}{2}+\frac{1}{3}$	5. Anticipate students' thinking. Students will need to understand and visualize equivalent fractions before trying this. Students should use manipulatives or drawings of number lines, area models, etc. in order to visualize the parts of the whole and the idea of combining the fractions. Students often choose to draw pie-shapes. Encourage them to try other pictures as well. Another option: Extend the depth further using strategy 1: Create a story. The higher complexity problem would only be for students who have already solved the simpler problem.
2. Choose a depth strategy. 8. Solve to learn. 2 Draw a picture. 3. Explain why.	
3. Apply the strategy. Use the same task, $\frac{1}{2}+\frac{1}{3}$, but give it to students before teaching the steps for adding fractions with unlike denominators. Have them make a drawing and use it to explain their thinking.	
4. Modify the complexity. Increase the size of one or both denominators so that students have to deal with more parts in the whole.	

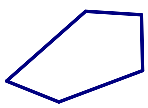
1. Identify a math task. 8 – 3	5. Anticipate students' thinking. Students may start by noticing that: • When the first number goes down 1, the difference goes down 1. • When the second number goes down 1, the difference goes up 1. • When both numbers go down 1, the difference stays the same. As they continue making the numbers go down by 1, they will begin to encounter negative numbers, and they can use the patterns they have seen in order to make predictions about these differences. It may be helpful for students to try picturing the differences on a number line. They are not expected to learn or understand rules for subtracting negative numbers, but they will begin to see that you can learn about new mathematical situations by starting with what you know and building patterns.
2. Choose a depth strategy. 10. Ask "What if…" 9. Build a pattern. 3. Explain why.	
3. Apply the strategy. What happens to the difference if: • The first number goes down 1? • The second number goes down 1? • Both numbers go down 1? Continue the pattern and explain why it happens.	
4. Modify the complexity. (1) Continue with different combinations of up and down. For example, "What if the first number goes up by 5 and the second number goes down by 2?" (2) Try expressions with more parts, such as 8 – 3 + 2, and continue to explore different combinations of up and down.	

Ten Plus One
Sample Tasks to Modify

Try your hand at enhancing the depth and complexity of these tasks!

Grade 1
1. What number is 10 less than 83?
2. True or False: 7 + 4 = 6 + 5
3. Which is longer? ________ _______

Grade 2
1. Write <, >, or =. 302 ____ 297
2. Count the sides and name the shape.

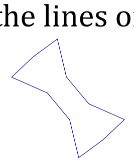


3. Write 528 in expanded form.

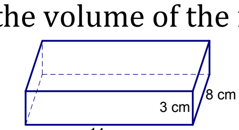
Grade 3

1. Round 293 to the nearest hundred.
2. 15 ÷ 3
3. Make the fractions equivalent. $\frac{2}{3}$ $\frac{6}{\square}$

Grade 4
1. Find the quotient and remainder. 352 ÷ 7
2. Find all of the factors of 42.
3. Draw the lines of symmetry.

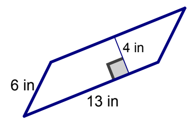


Grade 5
1. Write <, >, or =. 0.7 ___ 0.58
2. 8.4 • 1000
3. $6 \cdot \frac{2}{3}$
4. Find the volume of the rectangular prism.



Grade 6

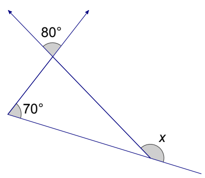
1. 18 is what percent of 40?
2. $\frac{1}{2} \div \frac{1}{3}$
3. Find the area of the parallelogram.



4. Find the mean, median, and range. 31, 27, 32, 65, 29

Grade 7

1. 13 – -5
2. A shirt that costs 16.50 is on sale for 20% off. What is the sale price?
3. Simplify. -5x – 3 + 9x – 1
4. Find the measure of $\angle x$.

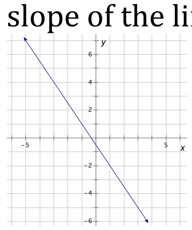


Grade 8

1. Decide if each number is rational or irrational:

$$\frac{3}{4} \quad \sqrt{10} \quad \frac{2.3}{0.8} \quad \pi \quad 0 \quad -6 \quad 2\cdot\sqrt{9}$$

2. Write 36,100 in scientific notation.
3. Find the slope of the line.



4. Find the length of the hypotenuse of a right triangle whose legs have lengths of 4 cm and 10 cm.