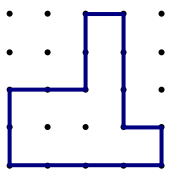
Grade 5 Monthly Problem Sets

These Monthly Problem Sets are designed to challenge gifted and talented math students in grade 5.
The problems are also meaningful and challenging for students in grades 6, 7, and even 8 who are working at or near grade level.

Notes and Suggestions:

- The problems are organized according to math content strands: *Number and Operation*; *Algebra*; *Geometry*; and *Data and Probability, Ratios and Rates*. There is also a *General* category that develops general problem-solving skills. Problems that address multiple strands appear under a heading for one of the relevant strands.
- Some problems have many solutions. Others may have no solution.
- When students solve a problem incorrectly, stress the importance of persistence!
- When students solve a problem correctly, encourage them to find another approach!
- Some problems anticipate standards that are typically met late in the school year. A few of them use concepts from grade 6 or 7 standards, especially if the concepts often appear in curriculum earlier than in the standards. Students can solve most of the problems using grade 5 knowledge, though doing so often requires some real creativity and persistence!
- The problems vary in difficulty. Be prepared—many of them, especially from the later sets, are very challenging and require a lot of time and determination. Students will typically not complete every problem on the page within one month. Remind them how much they can learn from a problem even when they are unable to solve it.
- Do not pre-teach rules and procedures just to help students solve a problem. Use the opportunity to encourage students to develop their own ideas and methods. If necessary, ask guiding questions or save the problem for later in the school year.
- Students should usually be able to work without direct instruction, but they may often get stuck. Encourage them to collaborate! Ask them to look up unfamiliar vocabulary. Check in with them from time to time to monitor their progress and offer encouragement.
- Unless otherwise directed, students may usually decide whether to use a calculator. However, they should be able to justify their decisions. Sometimes, they will learn a lot from solving a problem both with and without a calculator!
- Ask students to write and submit their thinking processes for some of the problems. Read their ideas carefully and offer brief comments. Your thoughtful comments are highly motivating for students.

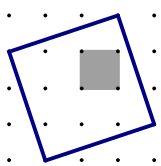
Grade 5
Monthly Problem Set 1

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	Jamilah's bank account has a balance of $248.32. In February she deposited $150.33, and wrote checks for $38.47, $101.00, and $13.17. What was her balance at the beginning of the month?	I am thinking of a number. If I add 2 to the number and multiply it by 4, the answer is 47 greater than the number. What number am I thinking of?	Decide if the following equations are true for the faces (F), edges (E), and vertices (V) of a pentagonal prism: F + V = E + 2 F + V – E = 3	Alicia has four tests in science class. She had scores of 83, 91, and 92 on the first three tests. What is the lowest score she can get on the final test in order to have a mean of 90?	If you travel only up and right, there are two paths from the lower left to the upper right in a 2 by 2 array of dots. Here is one path:  How many paths like this are there between the opposite corners of a 3 by 3 array of dots?
2	In this 3 by 3 *magic square*, every row, column, and diagonal has a sum of 15. 8 1 6 3 5 7 4 9 2 Make a 3 by 3 magic square for which each sum is 3.	Louise wants to buy a snack from a vending machine that takes quarters, dimes, and nickels. Write an algebraic expression for the cost of an item if she uses Q quarters, D dimes, and N nickels. How many combinations of coins can she use to buy a 45-cent snack?	How many square inches are in 3 square feet?	A student drops a thumbtack on the ground 200 times. It lands point up 81 times. Five other students repeat the experiment with results of 92, 98, 85, 93, and 103. Estimate the probability that the tack will land point up.	A diagonal of a polygon is a line segment that connects two vertices but is not a side of the polygon. How many diagonals does a hexagon have?
3	What is the greatest number in format 0.xxxx that equals 0.6 when rounded to the nearest hundredth and equals 0.595 when rounded to the nearest thousandth?	Leaving the numbers in the same order, but doing one operation at a time in any order you like, find as many different answers as you can to this expression. What do you get using the usual order of operations? 51 – 2 • 6 ÷ 3	The area of the polygon is 225 square units. Find its perimeter. 	The ratio of boys to girls at Jenny's party is 1 : 4. Two more people arrive and the ratio changes to 1 : 3. There are more than 15 guests. How many people are there?	A palindrome is a number that reads the same forward and backward. Jimmy's car odometer reads 47974. How many more miles must he drive before his odometer shows the next palindrome?

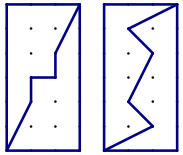
Grade 5
Monthly Problem Set 1 Solutions

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	$250.63	13 $(13 + 2) \cdot 4 = 60$ $13 + 47 = 60$ Most students use a guess, test, revise approach. Some may find a way to use algebra.	F = 7, E = 15, V = 10 The first equation is true. It is impossible for the second one to be true when the first one is.	94	6 paths You can write a "code" for each path using U for up and R for right. For example, the code for the path shown in the problem is RU.
2	Possible answer: 1.6 0.2 1.2 0.6 1 1.4 0.8 1.8 0.4 Students may write answers using decimals or fractions. One strategy: Divide each number in the original square by 5.	$0.25 \cdot Q + 0.1 \cdot D + 0.05 \cdot N$ Some students may omit the "·" symbol for multiplication. 8 combinations: 9N 1D, 7N 2D, 5N 3D, 3N 4D, 1N 1Q, 2D 1Q, 1D, 2N 1Q, 4N	432 square inches Hint: Draw a sketch of a square inch inside of a square foot!	About 46%	9 diagonals For concave polygons, the diagonals may extend outside the polygon.
3	0.5954	13, 98, and 47 Using the usual order of operations, the answer is 47.	80 units 225 ÷ 9 = 25. Each square has an area of 25 square units, so each side length is 5 units.	32 (8 boys and 24 girls)	110 miles Jimmy's odometer will read 48084.

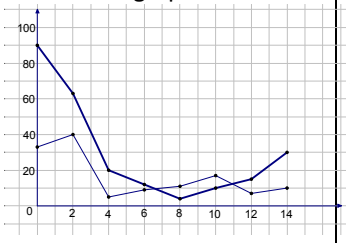
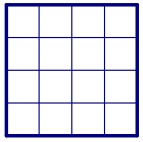
Grade 5
Monthly Problem Set 2

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	What is the sum of the numerator and denominator if you write 0.4375 as a fraction in simplest form?	Suppose that $a \Delta b$ means to add a and b, then double the answer. Find the values of $(3 \Delta 1) \Delta 4$ and $3 \Delta (1 \Delta 4)$. Is Δ an associative operation?	The grey square has an area of 1 square unit. Find the exact area of the tilted square. 	Five whole numbers have a mean of 3.8 and a median of 4. The only mode is 2. A number is added to the list so that the new mean is 5. What is the new median?	Find at least two creative ways to cut a rectangle into two congruent pieces.
2	Juan divided two whole numbers. His answer was 11 R 13. What two numbers might he have divided?	What value of a makes this equation true? $7 - 3 \cdot a = -0.5$	There are 640 acres in a square mile. A rectangular piece of land is half a mile long and an eighth of a mile wide. What is its area in acres? What is its area in square miles?	4 people can decorate 4 cookies in 4 minutes. How many cookies can 1 person decorate in 1 hour?	Which set of numbers could be the lengths of the sides of a triangle? Why? 2, 3, 5 3, 4, 8 4, 5, 7
3	Find a fraction between $\frac{1}{2}$ and $\frac{2}{3}$ with the smallest possible denominator.	Find at least five pairs of numbers that are solutions to $8 \div N = R \div 9$. What is the relationship between N and R? Suggestion: Make a table.	You open the bottom and top of an empty cereal box and cut off the flaps. What happens to the volume and the surface area of the box as you flatten it?	Two cars are 600 miles apart. They are driving toward each other, one at 55 mi/hr and the other at 65 mi/hr. How far apart will they be an hour and a half before they meet?	8 players joined a chess tournament. Every player played against every other player. How many games were played?

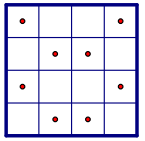
Grade 5
Monthly Problem Set 2 Solutions

	Number and Operation	Algebra	Geometry and Measurement	Data and Probability; Ratios and Rates	General
1	23 The fraction is $\frac{7}{16}$.	(3 Δ 1) Δ 4 = 24 3 Δ (1 Δ 4) = 26 Δ is not associative, because the value changes when you change the grouping of the numbers.	10 square units.	4.5	Two of many possibilities:  Students do not have to use dot or graph paper, but it may help!
2	Some possible answers: 167 ÷ 14 178 ÷ 15 189 ÷ 16 200 ÷ 17 Students might look for patterns in these answers! Another solution: 1113 ÷ 100	$a = 2.5$	40 acres $\frac{1}{16}$ of a square mile	15 cookies They must be doing some fancy decorating in order for it to take this long!	4, 5, 7 The sum of the lengths of the two shorter sides must be greater than the length of the longest side. Otherwise, the two shorter sides will not meet. Try it!
3	$\frac{3}{5}$ Notice that a common mistake in *adding* fractions leads to $\frac{1+2}{2+3}$, which is the correct answer to this question. Surprisingly, this is not a coincidence! Some students may like to explore this idea further.	Some possible solutions: $N = 1 \;\; R = 72$ $N = 2 \;\; R = 36$ $N = 4 \;\; R = 18$ $N = 8 \;\; R = 9$ $N = 16 \;\; R = 4.5$ $N = 0.5 \;\; R = 144$ The product of N and R is always 72!	Flattening the box decreases its volume but leaves the surface area the same.	180 miles The original distance between the cars is not needed to solve the problem.	28 Each player played seven people. 8 · 7 = 56. This counts each game twice because there are two people in each game. 56 ÷ 2 = 28

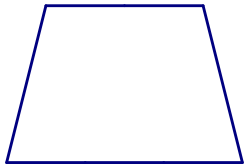
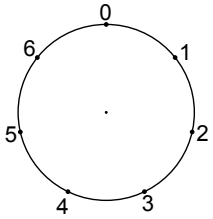
Grade 5
Monthly Problem Set 3

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	Raymond divided two whole numbers. The decimal part of his answer was .625. Donna found the quotient and remainder for the same two numbers. What was Donna's remainder?	Three vertices of a trapezoid are located at (1, 2), (1, 8), and (3, 9). Where can the fourth vertex be if both coordinates are whole numbers between 0 and 10 inclusive? (*Inclusive* means including 0 and 10).	Two vertices of a triangle have coordinates of (2, –1) and (3, 3). Find coordinates of a third vertex to create a triangle with an area of 14 square units. (If you find an answer and still have time, look for more!)	If seven whole numbers have a mean of 20 and a median of 10, what is their smallest possible range?	Each letter represents a digit. Find values for each letter to make the equation true. UP • TO = IT • GO
2	About how many square tiles would it take to cover a football field if each tile were 8 inches on a side? (Don't forget the end zones.) Just for fun, make a guess first!	You may rearrange –7 + 9 – 3 in many ways without changing the answer. For example: –3 + 9 – 7. Find as many ways as you can to do this using only addition and subtraction with the numbers 3, 7, 9, and their opposites –3, –7, and –9.	What do these capital letters have in common? One letter is missing. Which one? Explain. H, I, O, S, X, Z	If 8 popcorn vendors can sell 1080 bags of popcorn in 5 hours, how long will it take 9 popcorn vendors to sell 324 bags (if they all sell popcorn at the same constant rate)? (Inspired by a question about pretzel makers in *The Number Devil*, by Hans Magnus Enzensberger)	Multiply the smallest prime number greater than 100 by the greatest prime number less than 30. Divide the product by the number of millimeters in a meter. Take the whole number part of your answer and subtract the smallest prime number. What is the result?
3	Hope is making a pyramid of blocks. The top has 1 block. There are 3 blocks on the next layer, 5 blocks on the next layer, etc. If Hope uses 1000 blocks, how many layers tall is the pyramid? How many blocks are not used?	Make an input/output table for the expression. Find a pattern, and use it to write an algebraic equation that is true for (nearly) all values of *x*. $\frac{x^2 - 1}{x - 1}$ What is the one value of *x* for which your equation is not true?	A square is decomposed into five congruent rectangles, each having a perimeter of 30 centimeters. What is the area of the square? 	Create a title, labels, units, and a detailed story for the double-line graph. 	Place the greatest possible number of dots in the grid (one dot per square at most) so that every row, column, and diagonal has at most two dots. 

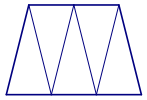
Grade 5
Monthly Problem Set 3 Solutions

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	It depends on what the divisor was. (Any positive multiple of 5 is a possible remainder.) Examples Divisor: 8 Remainder: 5 Divisor: 16 Remainder: 10 Divisor: 24 Remainder: 15 Divisor: 32 Remainder: 20 Divisor: 1000 Remainder: 625	There are 11 locations! Eight of them are easier: (3 ,0) (3, 1) (3, 2) (3, 4) (3, 5) (3, 6) (3, 7) (3, 8) The other three are trickier! (5, 4) (7, 5) (9, 6) Note: (3, 3) would make the figure a parallelogram, not a trapezoid.	Many answers are possible. Students may find (9, –1) or (2, 27), because the base and height are easier to see for these. Other answers include: (6, –13), (7, –9), (8, –5), (10, 3), (11, 7), etc. There are interesting patterns here that may help you predict some other solutions!	The smallest possible range is 24. The idea is to make the minimum as large as possible and the maximum as small as possible. The seven numbers that have this mean, median, and range are 10, 10, 10, 10, 33, 33, 34.	One solution is $14 \cdot 65 = 26 \cdot 35$. Can you find others?
2	129,600 tiles (if you cut some tiles) Students may need to look up the length and width of the field (360 ft by 160 ft). Their answer may be a little different if they overlapped the edge or left a gap along it, or if they rounded during their calculations.	There are many other possibilities. $-7 - 3 + 9$ $-3 - 7 + 9$ $9 - 3 - 7$ $9 - 7 - 3$ All of these expressions are equal to –1. Some students may use negative numbers in the second or third position: $-7 + -3 + 9$ or $-7 - 3 - -9$.	The letter "N" is missing. All of the letters have half-turn symmetry. In other words, they look the same after they are rotated 180° (turned upside-down).	1 hour 20 minutes	0 $101 \cdot 29 = 2929$ $2929 \div 1000 = 2.929$ $2.929 \rightarrow 2$ $2 - 2 = 0$
3	31 layers tall; 39 blocks left Some students may notice a pattern. When you find the sums of the blocks on the layers, you get square numbers. $1 + 3 = 2^2$ $1 + 3 + 5 = 3^2$ $1 + 3 + 5 + 7 = 4^2$, etc. ($31^2 = 961$)	The output is always one greater than the input! Therefore, the equation is $\frac{x^2 - 1}{x - 1} = x + 1.$ It is true for all values except $x = 1$, because the left side would be 0/0, which is not a number.	156.25 square centimeters Each side of the square has a length of 12.5 centimeters. (If students are unsure about multiplying decimals, they may create a picture on graph paper to help them *see* the number of square units.	Check that students' stories have a title, label, and units, that they make it clear what each line represents, and that the story fits the graphs. The horizontal label should be a time variable (second, hours, days, weeks, years, etc.)	This is one way to fit eight dots. Can you do better? 

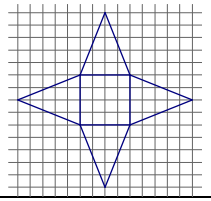
Grade 5
Monthly Problem Set 4

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	A recipe calls for one third of a cup of butter. A stick of butter contains half of a cup. What fraction of the stick do you need? Explain.	A rectangle's length is 4 times its width. The area of the rectangle is 361 cm^2. What are the dimensions of the rectangle?	Sketch a three-dimensional figure that has 7 faces, 7 vertices, and 12 edges. Name the figure if possible.	A 0.21-gram piece of candy corn contains 0.0028 g of sodium and 0.15 g of sugar. Write some whole number ratio comparisons. Express each ratio in simplest form.	Find a multiple of 11 whose digits sum to an odd number.
2	A rectangle's perimeter is between 21 and 22 units, and its area is between 29 and 30 square units. What are its dimensions?	$5 \cdot v + w = 7 \cdot v - w$ How does v compare to w?	Without leaving your seat, estimate the volume of your classroom.	The ratio of birds to dogs at a pet adoption facility is currently about 2 : 3. The ratio of dogs to cats is about 4 : 5. What is the approximate ratio of birds to cats?	How many different 3-digit whole numbers can you create using only the digits 1, 5, and 9?
3	$20 \cdot n = a \quad a \cdot n = 45$ What is the value of n?	Insert grouping symbols (parentheses or brackets) to make the equation true. $12 - 9 - 9 - 6 \cdot 1 + 8 - 20 = 10$	This is a scale drawing of an isosceles trapezoid with an area of 20 cm^2. Estimate its height. 	April found the mean of four numbers to be 11. Tom found the found mean of three other numbers to be 13. Patience predicts that the mean of all seven numbers is 12, because 12 is the mean of 11 and 13. Is she correct? Why or why not?	According to this "clock," $4 + 5 = 2$.  (1) $1 - 3 = ?$ (2) $2 \cdot 6 = ?$ (3) $3 \cdot ? = 1$ (4) $5 \div 2 = ?$ Note: All answers are numbers on the clock.

Grade 5
Monthly Problem Set 4 Solutions

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	Two thirds of the stick. (Students should explain their reasoning. There are many ways to think about this!)	Width: 9.5 cm Length: 38 cm	Any hexagonal pyramid satisfies the conditions.	Sample ratios of weights: sugar : total 5 : 7 sodium : total 1 : 75 sodium : sugar 7 : 375 Is there a quick way to find the last ratio from the first two?	The smallest number satisfying this condition is 209. The next is 308. Do you see a pattern? How long does it last?
2	There are many answers. Examples: 6 and 4.9 5.9 and 5 6.1 and 4.8 Students may learn more without a calculator. Have them estimate as part of their thinking process and to find more than one solution!	*v* and *w* must be equal! Students will probably discover this by trying many different values. Some may notice patterns and use them to explain *why* *v* = *w*.	Answers will vary. A 30- foot by 20-foot by 10-foot classroom has a volume of 6000 cubic feet. Students may use known lengths such as their own height to estimate the dimensions of the room.	about 8 : 15 (or approximately 1:2) Example: If there were 16 birds, there would be 24 dogs and 30 cats. The ratio 16 : 30 simplifies to 8 : 15. Some students may predict and test a quick way to find this answer!	27 different whole numbers Many students will create an organized list. Some may recognize that each place may contain one of three possible digits, which creates $3 \cdot 3 \cdot 3 = 3^3 = 27$ combinations.
3	$n = 1.5$ (Also, $a = 30$.) $20 \cdot 1.5 = 30$ and $30 \cdot 1.5 = 45$. Students may learn more if without a calculator. They do not need to know procedures for multiplying decimals (or fractions) in order to see that multiplying by 1.5 adds half of a number to itself.	$12 - (9 - ((9 - 6) \cdot (1 + 8) - 20))$ There may be other solutions.	The trapezoid's height is about 4 cm. This type of drawing may help:  (The height is about the same length as the shorter base of the trapezoid.)	No; the mean is $11\frac{6}{7}$. Students may experiment with many sets of numbers to check this. Think: Does it matter what the numbers are? Why is the mean slightly closer to 11 than to 13? Is there a way to find the mean of all the numbers without testing seven specific numbers?	According to the "clock": (1) $1 - 3 = \mathbf{5}$ (2) $2 \cdot 6 = \mathbf{5}$ (3) $3 \cdot \mathbf{5} = 1$ (4) $5 \div 2 = \mathbf{6}$ This type of "circular" arithmetic is known as *modular* arithmetic.

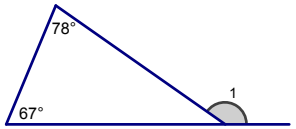
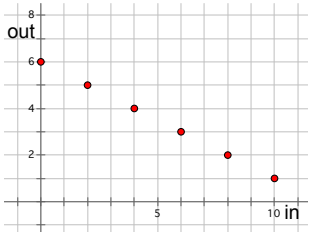
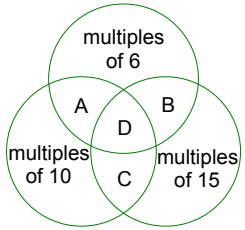
Grade 5
Monthly Problem Set 5

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	If you divide *m* by 7, the remainder is 3. If you divide *n* by 7 the remainder is 5. What happens when you divide *m* + *n* by 7? Explain.	Find at least 8 solutions to the equation: 5 – G = H – 2. Write each solution as an ordered pair in the form (G, H). What do you notice about all of your solutions?	The volume of a square prism is 80 cubic cm. Its length, width, and height are whole numbers. What is the largest possible surface area of the prism?	Javier's mean test score in math was 76 points. He receives 96 points on the next test and increases his mean to 80 points. How many tests has Javier now taken?	One of the interior angles in a parallelogram has a measure of 68°. What are the measures of the other interior angles?
2	Surabhi is watching the women's World Cup live on television in Denver. The game is being played in London with a local start time of 16:30. Surabhi watches for 1.6 hours before she has to leave for work. What time does her clock read when she leaves?	*Y* cars are in a line touching each other bumper to bumper. Write an algebraic expression that represents the number of bumpers that are touching another bumper.	This is a net for a three-dimensional figure whose base has an area of 100 square meters. Name the figure, and find its surface area. 	Towns A, B, C, and D lie in order on a straight road. AD (the distance from A to D) = 76 miles. Also, AB : BC = BC : CD AB : CD = 4 : 9 How far is town B from town D?	Jenny's can of soda holds 25% more soda than Dawn's. Dawn's can holds 20% less soda than Jenny's. How many ounces does each person's can hold?
3	Fill in the blanks in at least three different ways: 76.1 equals _____ tenths and _____ hundredths.	B 2 5 8 10 17 C 14 15.5 17 18 21.5 Write a rule that turns B into C. Then write a rule that turns C into B.	A prism has 14 vertices. How many faces and edges does it have?	Five friends have a mean number of 4 pets. One person has 10 pets. Which is greater, the mean or the median number of pets?	If every person in the United States could stand side by side at the equator, would they encircle the Earth?

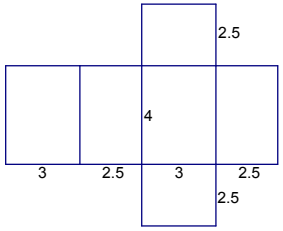
Grade 5
Monthly Problem Set 5 Solutions

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	The remainder will be 1. The two remainders combine to form 3 + 5, but this creates another group of 7, with 1 remaining.	Sample solutions: (0, 7) (1, 6) (2, 5) (3, 4) (4, 3) (5, 2) (6, 1) (7, 0). Any pair for which G and H have a sum of 7 works! This includes solutions in which G and H are fractions, decimals, or negative numbers.	322 square cm. The greatest surface area comes from a tall prism with a small base: in this case, 1 cm by 1 cm by 80 cm. Each base has an area of 1 square cm. Each of the other four faces has an area of 80 square cm.	Javier has now taken 5 tests. His total test score for the first four tests was $76 \cdot 4 = 304$ points. The 96 points from the fifth test raised his total to 400 points, giving a mean of $400 \div 5 = 80$ points.	68°, 112°, and 112° One of the angles has the same measure as the given angle. What is a quick way to find the measure of the other angle? Why does this make sense?
2	11:06 am Mountain Daylight time is 7 hours earlier than London time. 16:30 is 4:30 pm, so the game begins at 9:30 am in Denver. 1.6 hours is 1 hour and 36 minutes.	$(Y - 1) \cdot 2$ or $Y \cdot 2 - 2$ Encourage students to explain why their expressions make sense!	The figure is a square pyramid. Its surface area is 350 square meters.	Answer: BD = 60 miles Details: AB = 16 miles BC = 24 miles CD = 36 miles	It is not possible to say how many ounces are in each can. If Jenny's can holds 25% more than Dawn's, then Dawn's will *always* hold 20% less than Jenny's! Can you draw a picture to show why?
3	A few possible solutions: 761 tenths and 0 hundredths 0 tenths and 7610 hundredths 760 tenths and 10 hundredths 61 tenths and 7000 hundredths, etc. Some students may get creative!	Turning B into C: $C = B \div 2 + 13$ or $C = (B + 26) \div 2$ Turning C into B: $B = (C - 13) \cdot 2$ or $B = C \cdot 2 - 26$ Some students may use words instead of symbols to describe their rules.	The prism has 9 faces and 21 edges. (It is a heptagonal prism.)	The mean is greater. Because the mean number of pets is 4, the friends have a total of $4 \cdot 5 = 20$ pets. The other four friends have $20 - 10 = 10$ pets. If the median were 4, the sum of the two middle numbers would have to be 8, which would leave only 2 pets for the remaining two people.	Yes; they would encircle the Earth over 3 and one half times! This answer assumes that each person takes up an average width of 1.5 feet. (The current U.S. population is about 322,000,000 people. The circumference of the Earth is approximately 24,900 miles.)

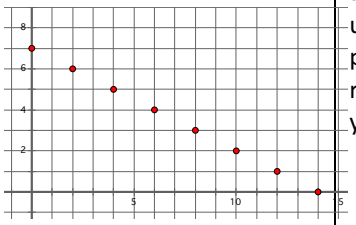
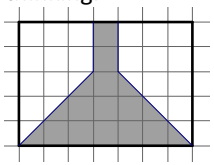
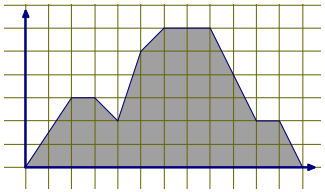
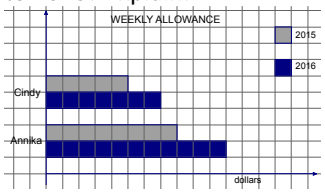
Grade 5
Monthly Problem Set 6

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	Does 34 ÷ 8 = 30 ÷ 7? Explain.	What is the relationship between T and V in this number sentence? Find at least one pair of solutions in which T and V are not both whole numbers. 45 ÷ T = 10 ÷ V Hint: make a table.	What is the measure of angle 1? 	Andrew drove 6200 miles in 8 months. If he continues to put miles on his car at the same rate, how many miles will he have driven after 2½ years?	What is the smallest square number that is a multiple of 63?
2	Jeanne charges $9.00 per hour to mow lawns. She mowed one lawn in 1 hour 15 minutes and another in 50 minutes. How much did she earn?	If Q ÷ 1.5 = 37, then what is the value of Q ÷ 15? Explain your thinking.	A rectangular prism has a width of 13 cm and height of 16 cm. How much does the volume change if the length of the prism increases by 0.5 cm?	A piece of carpet has an area of 96 square feet. The ratio of the length to the width is 3 : 2. How can you cut the same length from each side so that the ratio becomes 5 : 3?	64 is a square number and a cube number, because $8^2 = 64$ and $4^3 = 64$. Find a three-digit number that is a square number and a cube number.
3	Find the value of 7.51 hundredths minus 0.0038 tenths. Round your answer to the nearest thousandth. Write your rounded answer as a decimal and as a fraction in simplest form.	Create a table, a story, and a rule for this graph. 	Make a net for a rectangular prism that has a volume of 30 cubic inches and a surface area of 59 square inches.	On a county map, $1\frac{1}{4}$ inches represent 30 miles. How many miles does $\frac{3}{8}$ of an inch represent?	 What are the questions? What are the answers?

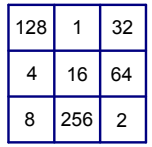
Grade 5
Monthly Problem Set 6 Solutions

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	They are both 4 r 2, but they are not equal, because the remainder of 2 stands for $\frac{2}{8}$ in the first expression and $\frac{2}{7}$ in the second one. (This explains why you should not write "=" for a quotient and remainder.)	T is always 4.5 times greater than V. Some possible solutions: T = 4.5 and V = 1 T = 9 and V = 2 T = 22.5 and V = 5 T = 45 and V = 10	145° Students need to know the sum of the interior angles in a triangle. Some may notice that 145 = 78 + 67. Challenge them to explain why this happens!	23,250 miles	441 441 is $3 \cdot 3 \cdot 7 \cdot 7$. It is the square of 21 ($3 \cdot 7$), and it is 7 times greater than 63 ($3 \cdot 3 \cdot 7$). (Knowing the prime factorization of 63 is helpful!)
2	$18.75	3.7 An easy way: the new divisor is 10 times greater, so quotient becomes one tenth as large.	The volume of the prism increases by 104 cubic cm regardless of the original length measurement of the prism!	Cut 2 feet from the length and the width. (The original length and width are 12 ft and 8 ft.)	729 $27^2 = 729$ and $9^3 = 729$. Notice that $(3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3) =$ $(3 \cdot 3) \cdot (3 \cdot 3) \cdot (3 \cdot 3)$
3	0.075, $\frac{3}{40}$ The calculation is 0.0751 – 0.00038. 0.07472 rounded to the nearest thousandth is 0.075. (Can you think of a quick way to understand why $\frac{3}{40} = 0.075$? Hint: Start with three fourths.)	Table: in 0 2 4 6 8 10 (days) out 6 5 4 3 2 1 ($) Possible story: You start with $6. Every two days, you pay your friend $1 of what you own her. Rule: The output is 6 minus half of the input; or A = 6 – D ÷ 2 where A is the amount of money you have after D days.	Possible answer (units are inches)  	9 miles Stress the importance of checking that results are reasonable as students work. Encourage students to share and compare strategies. There are *many* ways to find the answer! Was anyone able to think of it without shifting to decimals?	Question: What belongs in A, B, C and D? Region D contains multiples of 30. Regions A, B, and C are empty, because any number that is a common multiple of any pair from 6, 10, and 15 must also be a multiple of the third number! For example, A is empty because every common multiple of 6 and 10 is also a multiple of 15, placing it in region D.

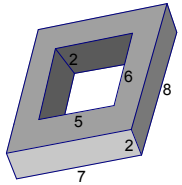
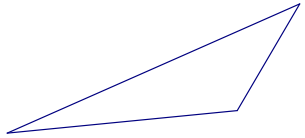
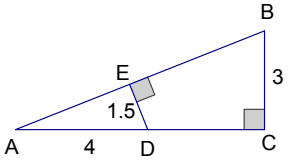
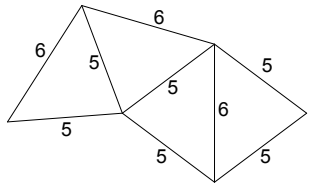
Grade 5
Monthly Problem Set 7

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	Add or subtract the fractions $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, and $\frac{1}{5}$ to create the smallest positive number you can. Use each fraction exactly one time.	Which of these expressions do you think are *equivalent* to each other? Why? $t + u - v$ $t - v + u$ $u - t + v$ Write another equivalent expression.	55 pennies are arranged on a flat surface in layers to make a figure that looks like a (filled) triangle. The top layer has one penny. Each layer has one more penny than the layer above it. Estimate the area of the triangle.	You have a standard deck of 52 cards. How can you place the cards in three piles so that if you choose a random pile and then a random card from that pile, you have the best chance of drawing a red card?	Complete the square so that each row, column, and main diagonal has the same product. 32 16 2
2	The area of a rectangular piece of carpet is 18 square feet. Its perimeter is 19.4 feet. What are the dimensions of the carpet?	Create a table, story, and formula for the graph. 	The volume of a prism whose base is the rectangle shown in bold is 165 cubic units. Find the volume of a prism having the shaded region as a base. Explain your thinking. 	What is the mean height of this mountain? Explain your thinking. (The side length of each small square on the grid represents 1000 feet.) 	Suppose a square computer chip measures 0.5 cm per side. Would 1,000,000 of these chips be too many, too few, or just about right to cover your classroom floor? Explain.
3	If $A \div 2.4 = 7.1$, then what is the value of $A \div 4.8$? Explain your thinking.	Find some pairs of (non-negative) values for which the inequality $x^2 + y^2 \geq (x + y)^2$ is true. Describe any discoveries that you make.	If possible, describe (or create or draw) a polyhedron having 8 faces and 6 vertices. Explain your thinking.	Whose allowance increased by a greater fraction of the original amount from 2015 to 2016? Explain. 	There were 381 M&Ms in a bag. Two mothers and their two daughters shared all of them equally between themselves. Each person received a whole number of M&Ms. How many M&Ms did each person receive?

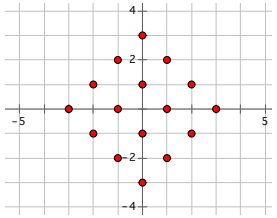
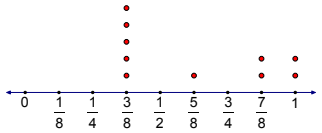
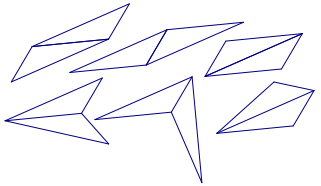
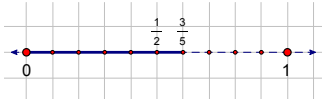
Grade 5
Monthly Problem Set 7 Solutions

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	Possible answers: $\frac{1}{2}+\frac{1}{5}-\frac{1}{3}-\frac{1}{4}=\frac{7}{60}$ $\frac{1}{2}-\frac{1}{4}+\frac{1}{5}-\frac{1}{3}=\frac{7}{60}$ How many others ways can you rewrite this expression to give the same result?	The first two expressions are equivalent, because they always give the same answer as each other, no matter what numbers you substitute for *t*, *u*, and *v*! Some other expressions that are equivalent to these two are *u* – *v* + *t* and *u* + *t* – *v*.	Sample answer: 28 in^2 (181 cm^2) The triangle has 10 pennies on the base and is 10 pennies tall. Since each penny is ¾ of an inch in diameter, the base and height are each about 7.5 inches. 7.5 · 7.5 ÷ 2 = 28.125 sq. in. Why might peoples' answers be different? Does the pennies' arrangement matter?	Place one red card in one pile, another red card in another pile, and the remaining 50 cards in the third pile. If you draw from either of the first two piles, you are certain to draw a red card. If you draw from the third pile, you have slightly less than a 50% chance of drawing a red card. (Overall, your chances are slightly less than $\frac{5}{6}$.)	 Notice that each number in the square is a *power of two*. What happens if you take half of every number in the square?
2	Length: 7.2 feet Width: 2.5 feet The names "length" and "width" are interchangeable.	x 0 2 4 6 8 10 12 14 y 7 6 5 4 3 2 1 0 Sample story: Janine begins with 7 candy bars. Every two days, she eats one bar. How many candy bars does she have left after each two-day period? Some possible equations: y = 7 – 0.5 · x y = 7 – x ÷ 2	The volume is 66 cubic units. The area of the shaded region is 4 tenths (40%) of the area of the rectangular base. The volume of the prism is the same fraction of 165: 165 ÷ 10 · 4 = 66 Note: It is not necessary to calculate a height (though many students will probably choose a strategy that includes this step)!	The mean height is 3,375 feet. The mean is not just about adding up numbers and dividing. If the mountain were leveled out to a constant height, it would be 3,375 feet tall. The total area is 40.5 square units on the grid. Since the base is 12 units long, the height of a rectangle would be 40.5 ÷ 12 = 3.375 units, which represents 3,375 ft.	Too few. There are 4 computer chips in 1 square cm. Since 1 foot is about 30 cm, 1 square foot is about 900 square cm. Thus, it contains about 900 · 4 = 3600 chips. 1 million chips covers about 1,000,000 ÷ 3600 or 278 square feet, not enough for a classroom of about 30 feet by 20 feet, or 600 square feet. The chips might cover a typical bedroom floor, though!
3	3.55 Some students may multiply 7.1 by 2.4 and then divide the result by 4.8. However, since doubling the divisor makes the quotient half as great, all you need to do is to divide 7.1 by 2!	The inequality is true if *x* = 0, or *y* = 0, or both *x* and *y* are equal to 0. In these cases, the left and right sides are equal. The left side is never greater than the right side when both *x* and *y* are positive numbers.	You can create a polyhedron that fits the conditions by joining two rectangular pyramids at their bases. It helps to use triangles for faces in order to limit the number vertices. Are there other polyhedrons that work?	Cindy's allowance increased by a greater fraction. Hers increased by $\frac{2}{5}$ (or 40%) of the original amount, while Annika's increased by $\frac{3}{8}$ (or 37.5%) of the original amount, and $\frac{2}{5}>\frac{3}{8}$.	127 M&Ms. Since 381 is not divisible by 4, there must not have been 4 people. One person must have been both a daughter and a mother to others there. In other words, there is a girl, her mother, and her grandmother. 381 ÷ 3 = 127.

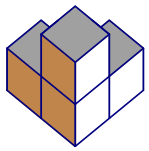
Grade 5
Monthly Problem Set 8

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	Find the sum: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots+\frac{1}{1024}$ (The dots mean to continue the pattern until you reach the final addend.)	Fiona Frog sits at the origin of a rectangular coordinate system. She jumps three times, randomly choosing between 1 unit left, right, up or down each time. How many different places can she land? Name their coordinates, and plot the points.	All faces of this figure except the top and bottom are rectangles. The units are centimeters. Find the volume and surface area. 	Using exactly ten "eighths" fractions between (possibly including) 0 and 1, create a line plot whose median is exactly halfway between the mode and the mean. What could the line plot be about?	How many different quadrilaterals can you make by joining two copies of this triangle side to side? Do any of the quadrilaterals have other names? 
2	Brookview Elementary school is having a field trip. Someone mistakenly reserves 15 school buses, which leaves the school 90 seats short. After quickly ordering 3 more buses, there are 39 empty seats. All of the buses have the same number of seats. How many people are riding the buses, and how many seats are on each bus?	Suppose that $a + b = c$ $b - c = d$ $c + d = e$ $d - e = f$. What is the value of f if $a = 3$ and $b = 4$? What is the value of f if $a = 21$ and $b = 39$? What is the pattern?	If AD = 4 in, BC = 3 in, and ED = 1.5 in, then find the length AB. 	Amanda, Caden, and Juan started eating a bowl of 420 pieces of popcorn at the same time. Amanda ate $\frac{2}{3}$ as fast as Caden, and Juan ate $2\frac{1}{2}$ times faster than Amanda did. How fast did Caden eat compared to Juan? (At what average) rate did each person eat if they finished the bowl in 6 minutes?	The difference of the squares of two consecutive numbers is 135. What are the numbers? The difference of the squares of two consecutive numbers is 270. What are the numbers?
3	Penny is going to visit her friend who lives $\frac{3}{5}$ of a mile from her. She has traveled $\frac{1}{2}$ a mile so far. What fraction of the distance has she traveled? Draw a number line that supports your answer.	Create a story to fit the equation $n = x + 2 \bullet y + 3 \bullet z$. Be sure to explain clearly what each variable represents.	Copy the figure, and join two more of these triangular faces to complete a net. Describe the 3D shape that it will make. Count the number of faces, vertices, and edges. Estimate or calculate the surface area. 	Find the value of V. $1\frac{1}{3} : 6 = V : 9$	If you subtract 2 from 10 times a number, the answer is the same as when you add 1 to 4 times the same number. What is the answer if you subtract 6 from 12 times the number?

Grade 5
Monthly Problem Set 8 Solutions

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	The sum is $\frac{1023}{1024}$. Students may notice patterns: • The denominators are powers of 2. • The numerator in each partial sum is always one less the denominator. • Each time you add another fraction, the answer gets closer to 1, but it will never reach 1! There may be many other patterns!	Fiona Frog can land in 16 locations: (3,0) (–1,2) (1,2) (–2,1) (0,1) (2,1) (–3,0) (–1,0) (1,0) (3,0) (–2,–1) (0,–1) (2,–1) (–1,–2) (1,–2) (–3,0).  What if she jumps 4 times? 5 times?	Volume: 52 cubic centimeters Surface area: 156 square centimeters.	A possible solution:  In this graph, the mode is $\frac{3}{8}$, the median is $\frac{1}{2}$, and the mean is $\frac{5}{8}$. The line plot could be about the fraction of shots made by ten people shooting 8 free throws each. (It might also be about something measured in inches, cups, etc.)	Six  The top three quadrilaterals are parallelograms. The others are kites. (You can create the top three shapes by rotating the triangle and bottom three from reflecting it.)
2	There are 735 riders and 43 seats per bus. Sample strategy: Adding three more buses changes the situation from a 90-seat deficit to a 39-seat surplus, which is a change of 90 + 39 = 129 seats. These seats are accounted for by three new buses, so each bus must have 129 ÷ 3 = 43 seats. 18 buses hold 18 · 43 = 774 people, but 39 of the seats are empty, so there must be 774 – 39 = 735 riders.	$f = -7$ when $a = 3$ and $b = 4$. $f = -60$ when $a = 21$ and $b = 39$. f will always equal the opposite of the sum of a and b. Some students may be able to explain why this happens.	AB = 8 inches Hint in case students have been stuck for a while: Draw the segment $\overline{BD}$, and look at triangle ABD. (Find bases and heights for it.) Triangle ABD has a base of 4 in and a height of 3 in, so its area is 3 · 4 ÷ 2 = 6 in^2. However, if AB is its base, then the height of triangle ABD is 1.5 in. In order for the area to be 6 in^2, AB must equal 8 in (so that 8 · 1.5 ÷ 2 = 12 ÷ 2 = 6).	Caden ate $\frac{3}{5}$ as fast as Juan. Amanda ate 14 pieces per minute. Caden ate 21 pieces per minute. Juan ate 35 pieces per minute. These are called *average* rates in the problem, because the kids probably did not eat at constant rates. There are *many* ways to solve this problem. Encourage students to share their strategies.	$68^2 - 67^2 = 4624 - 4489 = 135$ There is no solution to the second question. (The square of an even number is even; the square of an odd number is odd; and difference between an odd number and an even number is always odd.) There is an interesting pattern for the first part! Could you use it to find the answer more easily? $2^2 - 1^2 = 3$ $3^2 - 2^2 = 5$ $4^2 - 3^2 = 7$ $5^2 - 4^2 = 9$ $6^2 - 5^2 = 11$ etc.
3	Penny has traveled $\frac{5}{6}$ of the distance. You can show this is with a number line labeled in tenths.  If you think of $\frac{3}{5}$ as 1 whole, then $\frac{1}{2}$ is $\frac{5}{6}$ of that whole.	A sample story: If x is the number of free throws, y is the number of 2-point shots, and z is the number of 3-point shots made by a basketball team during a game, then n is the number of points scored. It is important to remember about the order of operations!	Hints: (1) Fold your net to be sure it works. (2) The height of each face is a whole number. You can find it by making an accurate drawing. The 3D shape: two triangular pyramids joined at their bases (which are equilateral triangles). There are 6 faces, 5 vertices, and 9 edges. The height of each triangle is 4 units, so the area of each triangle is 6 · 4 ÷ 2 = 12 square units. The surface area is 12 · 6 = 72 sq units.	$V = 2$ Two of many strategies: • 9 is 1.5 times greater than 6. What is 1.5 times greater than $1\frac{1}{3}$? Half of $1\frac{1}{3}$ is $\frac{2}{3}$. Add this to $1\frac{1}{3}$ to get 2. • Multiply $1\frac{1}{3}$ and 6 both by 3 to get the equivalent ratio 4 : 18, which simplifies to 2 : 9.	0 The number is ½, because 2 subtracted from 10 times the number is 5 – 2 = 3, and 1 added to 4 times the number is 2 + 1 = 3. Therefore, 6 subtracted from 12 times the number is 6 – 6 = 0.

Grade 5
Monthly Problem Set 9

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	Charissa read $\frac{1}{3}$ of her book yesterday and $\frac{1}{5}$ of it today. She still has 91 pages left to read. How many pages long is the book?	4 blaps equal 6 glips, and 5 clops equal 8 glips. How many blaps equal 3 clops?	A 5-foot-wide sidewalk surrounds a rectangular park. The park's area with the sidewalk is 95,200 square feet. Without the sidewalk, the area is 89,100 square feet. What is the outside perimeter of the sidewalk?	Ricardo rides his bike 5 miles in 24 minutes at a steady rate. What is his speed in miles per hour?	Find two numbers whose difference is 12 and whose greatest common factor is 9.
2	$\frac{5}{6} - \frac{\square}{\square} \; ? \; \frac{1}{9}$ Enter a one-digit numerator and a one-digit denominator to make the difference as close to $\frac{1}{9}$ as possible. Replace "?" by <, >, or =.	What is the largest number that makes this number sentence true? $126 - d > 45.1$	The blocks in this figure are cubes. The total volume of the figure is 108 cubic cm. What is its surface area? 	The area of the rectangle on the left is 50 square meters. Estimate the area of the rectangle of the right. Explain your thinking. 	Begin with a two-digit prime number. Reverse the digits to create another prime number. What is the greatest possible difference between the two numbers?
3	Find two fractions between $\frac{5}{6}$ and $\frac{6}{7}$.	Use all four numbers 1, 2, 3, 4 and the operations +, –, • and ÷ to create expressions that equal the whole numbers 11 – 20. Use each number only once in each expression. Remember to think about order of operations.	A rectangular prism has a volume of 12.5 cubic yards. What is its volume in units of cubic feet?	Ari has taken five math tests with a mean score of 72.6. When her teacher drops her lowest score, her mean increases to 82.25. What was her lowest test score?	Desmond is reading a mystery. Jeanne asks him what page he is on. He replies that the product of the facing pages is 11,556, and he is reading the page on the left. What page is Desmond reading?

Grade 5
Monthly Problem Set 9 Solutions

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	195 pages Charissa has read $\frac{8}{15}$ of the book, so there is $\frac{7}{15}$ left to read. $\frac{1}{15}$ of the book is 91 ÷ 7 = 13 pages. The entire book is 13 · 15 = 195 pages.	3.2 blaps are equal to 3 clops.	1240 feet The dimensions of the park with the sidewalk are 340 feet by 280 feet.	12.5 miles per hour	There is no solution. If the greatest common factor of the numbers is 9, then their difference must be 9, 18, 27, etc. (a multiple of 9).
2	The best answer is $\frac{5}{6}-\frac{5}{7}>\frac{1}{9}$ The next best answer is $\frac{5}{6}-\frac{3}{4}<\frac{1}{9}$.	There is no largest number that makes the number sentence true! 80.8 makes it true, but so do 80.89, 80.899, 80.8999, etc.! (80.9 does not make it true, because it makes the two expressions equal.)	162 square cm Each cube has a volume of 108 ÷ 4 = 27 cubic cm. Therefore, each side of a cube is 3 cm, and each face has an area of 9 square cm. The figure has 18 exposed faces. The total surface area is 9 · 18 = 162 sq cm.	Possible estimate: 33 sq. units. The rectangle on the right is one third as long and twice as tall as the one on the left. $50 \div 3 \cdot 2 = 33\frac{1}{3}$ Students' estimates may vary.	The greatest possible difference is 54 (from 71 – 17). Other pairs are 11, 11; 13, 31; and 37, 73. These have differences of 0, 18, and 36 respectively. (Note: 91 is not a prime number.)
3	There are infinitely many answers! A few of them are: $\frac{71}{84}$, $\frac{21}{25}$, and $\frac{11}{13}$. (There are also many strategies. Students should discuss what they did!)	2 · 3 + 1 + 4 = 11 4 · 3 · (2 – 1) = 12 4 · 3 + (2 – 1) = 13 4 · 3 + 2 · 1 = 14 4 · 3 + 2 + 1 = 15 4 · 2 · (3 – 1) = 16 (4 + 2) · 3 – 1 = 17 (4 + 2) · 3 · 1 = 18 (2 + 3) · 4 – 1 = 19 (2 + 3) · 4 · 1 = 20 Other solutions are possible. What happens if you are allowed to use exponents?	Volume: 337.5 cubic feet Each cubic yard contains 27 cubic feet. (Ask students to draw a picture that shows why this is true.) Does it matter that the figure was a rectangular prism?	34 points Ari earned a total of 72.6 · 5 = 363 points on all five tests. After the lowest score was dropped, she had a total of 82.25 · 4 = 329 points. The difference between these totals must be the number of points on her worst test: 363 – 329 = 34 points.	Desmond is reading page 107. 107 · 108 = 11,556.