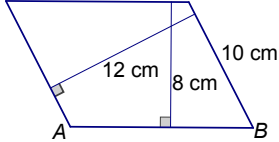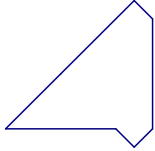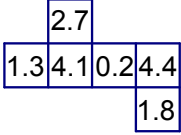# Grade 6 Monthly Problem Sets

These Monthly Problem Sets are designed to challenge gifted and talented math students in grade 6.
The problems are also meaningful and challenging for students in grades 7 and 8 who are working at or near grade level.
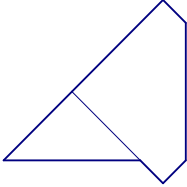
Notes and Suggestions:

- The problems are organized according to math content strands: *Number and Operation*; *Algebra*; *Geometry*; and *Data and Probability, Ratios and Rates*. There is also a *General* category that develops general problem-solving skills. Problems that address multiple strands appear under a heading for one of the relevant strands.

- Some problems have many solutions. Others may have no solution.

- When students solve a problem incorrectly, stress the importance of persistence!

- When students solve a problem correctly, encourage them to find another approach!

- Some problems anticipate standards that are typically met late in the school year. A few of them use concepts from grade 7 or 8 standards, especially if the concepts often appear in curriculum earlier than in the standards. Students can solve most of the problems using grade 6 knowledge, though doing so often requires some real creativity and persistence!

- The problems vary in difficulty. Be prepared—many of them, especially from the later sets, are very challenging and require a lot of time and determination. Students will typically not complete every problem on the page within one month. Remind them how much they can learn from a problem even when they are unable to solve it.

- Do not pre-teach rules and procedures just to help students solve a problem. Use the opportunity to encourage students to develop their own ideas and methods. If necessary, ask guiding questions or save the problem for later in the school year.

- Students should usually be able to work without direct instruction, but they may often get stuck. Encourage them to collaborate! Ask them to look up unfamiliar vocabulary. Check in with them from time to time to monitor their progress and offer encouragement.

- Unless otherwise directed, students may usually decide whether to use a calculator. However, they should be able to justify their decisions. Sometimes, they will learn a lot from solving a problem both with and without a calculator!

- Ask students to write and submit their thinking processes for some of the problems. Read their ideas carefully and offer brief comments. Your thoughtful comments are highly motivating for students.
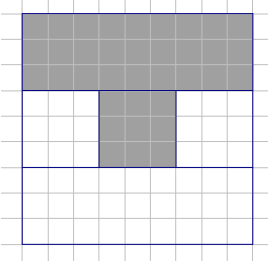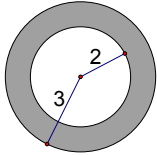
Grade 6
Monthly Problem Set 1

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	Is $\frac{2}{3}$ of $\frac{1}{2}$ of 9 greater than, less than, or equal to 3?	Find a rule for the input / output table. Write the rule as an algebraic equation. in (x) out (y) 3 5 6 32 7 45 9 77	What is the length of $\overline{AB}$ in the parallelogram below? 12 cm 10 cm 8 cm A B	Janine slept from 9:20 pm until 5:35 am. She is making a circle graph of how her time was spent in a 24-hour period. Estimate, and then calculate the number of degrees in the "sleep" portion of her graph.	If -5 is the 1st number in this sequence, what is the 87th number? -5, 3, 11, 19, 27 ... Extra: Find an algebraic expression for the nth number in the sequence.
2	Fatima read 30% of her book yesterday and 40% of the remaining pages today. She has 84 pages left to read. How many pages does the book have?	Give an example to prove that division is not an associative operation.	Show how to make one cut so that the two pieces may be assembled to form a square.	A *sample space* is the collection of all possible outcomes of a probability experiment. Find an organized way to list every member of the sample space for rolling two six-sided dice. How many outcomes are there?	Insert plus and minus symbols to create the number 363. Do it again to create the number 1. 1 2 4 3 8 7 9 5 6
3	An aquarium weighs 44 kg when it is half full with water and 64 kg when it is three-fourths full. How much does the aquarium weigh when it is empty?	Show a method to do this calculation mentally. Which property are you using (associative, commutative, or distributive)? Explain. $47 \cdot 86 - 47 \cdot 84$	Find the largest and smallest sum formed by three faces that meet at the same vertex when this net is folded to from a cube. 2.7 1.3 4.1 0.2 4.4 1.8	There is 50% probability of rain on Saturday and a 50% probability of rain on Sunday. What is the probability that it will rain over the weekend?	Use exactly four 3s and no other digits to create the number one-eighth. You may use the four usual operations along with exponents, fraction bars, decimal points, negative signs, percent symbols, parentheses, etc.

Grade 6
Monthly Problem Set 1 Solutions

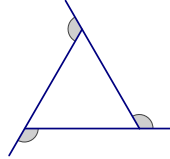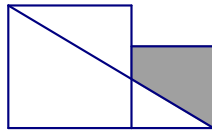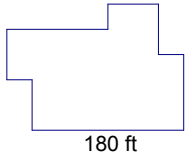|	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	It is equal to 3.	$y = x^2 - 4$ or $y = (x + 2) \cdot (x - 2)$ Other answers are possible.	15 cm	Estimate: a little over $\frac{1}{3}$ of the circle, or 120°. Answer: About 124° (123.75)	683 Extra: $-5 + 8 \cdot (n - 1)$ or $8 \cdot n - 13$
2	200 pages	Sample solution: $(10 \div 4) \div 2 = 2.5 \div 2 = 1.25$, but $10 \div (4 \div 2) = 10 \div 2 = 5$. Students may occasionally find a case where the answers are the same. However, the answer must *always* be the same for the operation to be associative.	Hint: It may be easier to see the cut if you rotate the shape 45°. 	1,1 1,2 1,3 1,4 1,5 1,6 2,1 2,2 2,3 2,4 2,5 2,6 3,1 3,2 3,3 3,4 3,5 3,6 4,1 4,2 4,3 4,4 4,5 4,6 5,1 5,2 5,3 5,4 5,5 5,6 6,1 6,2 6,3 6,4 6,5 6,6 There are 36 outcomes. Some students may have other ways to do this, including tree diagrams.	$12 - 438 + 795 - 6 = 363$ $1 + 2 - 4 + 3 + 8 - 7 + 9 - 5 - 6 = 1$ How many other ways can you find to make 1?
3	4 kg	$86 - 84 = 2 \quad 2 \cdot 47 = 94$ The value is 94. This strategy uses the *distributive* property, which shows how multiplication and subtraction (or addition) work together. $47 \cdot 86 - 47 \cdot 84 =$ $47 \cdot (86 - 84)$	Largest sum: 8.4 Smallest sum: 6.1	75%. This assumes that the probabilities are *independent.* In other words, what happens one day does not affect the probability for the other day.	Possible answers: $3 \div (3^3 - 3)$ $.3 \div (3 - .3 - .3)$ There are other ways!

Grade 6
Monthly Problem Set 2

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	Explain how to find the value of $2^4 \cdot 5^4$ as quickly and easily as you can without a calculator.	A department store has a sale for 30% off the original price of a shirt. Choose a variable and write an algebraic expression for the total cost of the shirt.	Find the circumference of a circle that has the same area as a square with side lengths of 1.5 meters.	Estimate the solution. Then calculate the exact value of u. $$\frac{8}{u} = \frac{12}{25}$$	A palindrome is a number such as 56865 that reads the same forward and backward. How many three-digit palindromes are divisible by 3?
2	What percentage of the square is shaded?	Suppose you place n cards face up in a row. Write an algebraic expression for the number of cards you will need to make a border with face down cards.	A rectangle with whole number side lengths has an area of 156 square centimeters. What is the greatest possible perimeter? What is the smallest possible perimeter?	Once per day, customers at the Ace of Hearts coffee shop may draw a card at random from a standard deck of 52 playing cards. They earn a point toward a free coffee if the card is an ace or a heart. 10 points earns a free coffee. If Osa comes every day, about how long do you think it will take her to win a free drink?	The greatest common factor (GCF) of a and b is 12. The least common multiple (LCM) of a and b is 120. Find the values of a and b.
3	Find the value of 0.1 ÷ 0.01. Explain why your answer makes sense.	$f \div g = 16$ If $x = f \div 2$ and $y = g \cdot 10$, then what is the value of $x \div y$?	What are the answers to the question above if the sides need not have whole number lengths?	What is the ratio of the area of the shaded ring to the area of the smaller circle? (Units are centimeters.)	Is this expression greater or less than 1? Write it as a simple fraction (with whole numbers in the numerator and denominator). $$\frac{\frac{1}{3}}{\frac{1}{2}}$$

Grade 6
Monthly Problem Set 2 Solutions

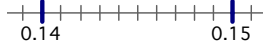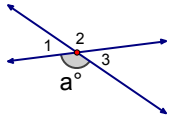|	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	Use the commutative and associative properties to rewrite $(2 \cdot 2 \cdot 2 \cdot 2) \cdot (5 \cdot 5 \cdot 5 \cdot 5)$ as $(2 \cdot 5) \cdot (2 \cdot 5) \cdot (2 \cdot 5) \cdot (2 \cdot 5)$ which equals 10^4 = 10,000.	If P represents the cost of the shirt, then possible algebraic expressions are $P - 0.3 \cdot P$ or $0.7 \cdot P$.	Approximately 5.3 meters.	The solution is slightly greater than 16 (so that the fraction will be a little less than $\frac{1}{2}$.) The exact value is $16\frac{2}{3}$.	30 three-digit palindromes
2	A little more than 44% of the square is shaded. (The exact answer is $44.\overline{4}$% or $44\frac{4}{9}$%.)	$2 \cdot n + 6$ or $2 \cdot (n + 3)$	Smallest perimeter: 50 cm (for a 13 by 12 rectangle) Largest perimeter: 314 cm (for a 156 by 1 rectangle)	A little over one month ($16 \div 52 \approx 0.31$ points per day. $10 \div 0.31 \approx 32$ days.) The actual time that it takes may vary a lot!	There are two answers: 12 and 120 24 and 60 (It does not matter which number in a pair is a or b.)
3	$0.1 \div 0.01 = 10$ because there are 10 groups of 0.01 in 0.1. Putting it another way, the quotient is always 10 when the dividend is 10 times greater than the divisor.	0.8 When you divide f by 2, the quotient becomes half of what it was. When you divide g by 10, the quotient becomes 1 tenth of what it was: ($16 \div 2 \div 10 = 0.8$). Some students may find the answer simply by trying different numbers for f and g.	Smallest perimeter: slightly less than 50 cm ($4 \cdot \sqrt{156}$). Largest perimeter: There is no limit to how large the perimeter can be! (Make the rectangle very "long and skinny.")	$5 : 4$, 5 to 4, $\frac{5}{4}$, or 1.25 (Students may look up the formula for the area of a circle if necessary.)	Less than 1, because $\frac{1}{3} < \frac{1}{2}$. Simple fraction: $\frac{2}{3}$. (Draw a picture to see that $\frac{1}{3}$ is $\frac{2}{3}$ of $\frac{1}{2}$. Or multiply the numerator and denominator by 6.)

Grade 6
Monthly Problem Set 3

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	Create a story problem for 6 ÷ ½. Explain why it makes sense that the quotient is 12.	Find the values of U, V, and W. U + V = 10 V + W = 7 U + W = 4	Find the sum of the *exterior* angles of an equilateral triangle.	If you roll two fair dice, what is the probability that the product of the numbers will be prime?	After 100, what it is the smallest three-digit number having an odd number of factors?
2	Jack bought a bike for $215. The bike was on sale from its usual price of $250. By what percentage was the price reduced?	Find a rule or formula that turns x into y. in (x) out (y) 0 0 6 4 10 6 and 2/3 21 14 150 100	The two squares have areas of 36 and 16 square units. Find the area of the shaded trapezoid.	Lindsay walks at a steady rate to meet her plane at the airport. For the final $\frac{2}{3}$ of the walk, she takes a moving walkway that doubles her pace. She arrives in 8 minutes. The walkway is out of order when she returns. How long does her return walk take?	What is the ones digit of $3^{43} + 6^{28}$?
3	Estimate the coordinates of the three points, and write at least two true equations that contain each number exactly once. (Use number sense instead of a calculator!)	Find the missing value, and write an algebraic expression for $a \neg b$ using familiar operations. Is $\neg$ a commutative operation? Explain. 4 ¬ 3 = 7 21 = 5 ¬ 2 56 = 9 ¬ 5 2 ¬ 4 = −12 -19 = 9 ¬ 10 6 ¬ 6 = 0 ? = 11 ¬ 8	A flat-roofed store has a height of 20 feet. The floor looks like this. 180 ft Estimate the store's volume and surface area.	Freya and Gitte are roommates. Freya earns $1800 per month and Gitte earns $2100 per month. They plan to share expenses proportionally to their income. If their total rent and utilities are $1100 per month, and their combined food costs for the month are $525, how much should each person pay for each expense?	A ball is dropped. Each time it bounces, it returns to half its previous height. The ball bounces four times and returns to the ground before someone stops it. If the ball has travelled a total of 26 feet 10 inches, how high was it when it was dropped?

Grade 6
Monthly Problem Set 3 Solutions

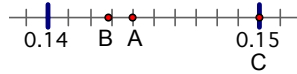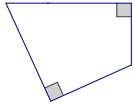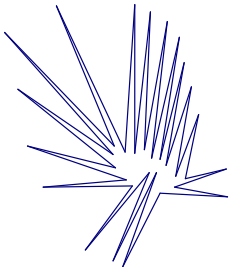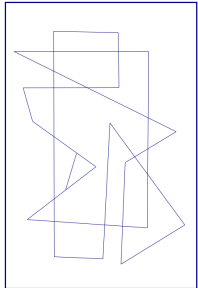|	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	Story: You have 6 cups of apple juice. If each person gets $\frac{1}{2}$ of a cup, how many people can you serve? $6 \div \frac{1}{2} = 12$, because there are 12 groups of $\frac{1}{2}$ in 6 (In other words, 6 is 12 times greater than $\frac{1}{2}$).	U = 3.5 V = 6.5 W = 0.5	360° (120° + 120° + 120°)	$\frac{1}{6}$ or approximately 16.7% There are 36 outcomes in the sample space. 6 of these are in the desired event {1, 2; 2, 1; 1, 3; 3, 1; 1, 5; 5, 1}	121 (Its only factors are 1, 11 and 121.) Square numbers)and square numbers only) have an odd number of factors. Can you see why?
2	The price was reduced by 14%.	There are many possibilities! $y = x \div 3 \cdot 2$ $\quad$ $y = x \cdot 2 \div 3$ $y = x \cdot \frac{2}{3}$ $\quad\quad$ $y = x \div 1.5$	11.2 square units	12 minutes (Notice that Lindsey spends the same amount of time on each part of her walk to the plane!)	3 Look at patterns in the ones digits for powers of 3 and powers of 6.
3	0.8, 0.1, and 0.08 (from right to left) are reasonable values that are practical to work with. Three possible equations: $\quad$ 0.8 · 0.1 = 0.08 $\quad$ 0.08 ÷ 0.8 = 0.1 $\quad$ 0.08 ÷ 0.1 = 0.8	? = 57 $a \neg b = a^2 - b^2$, or $a \neg b = (a + b) \cdot (a - b)$ Δ is not a commutative operation because $a \neg b$ does not always equal $b \neg a$. (In fact, $a \neg b$ is always the opposite of $b \neg a$. Mathematicians call it *anti-commutative*!)	Volume: about 486,000 cu ft Surface area: about 63,000 sq ft Students' answers will vary since they are estimates. (Note: The surface area includes the floor and the ceiling.)	Freya: $507.69 for rent and $242.31 for food. Gitte: $592.31 for rent and $282.69 for food.	9 feet 4 inches (or 112 inches)

Grade 6
Monthly Problem Set 4

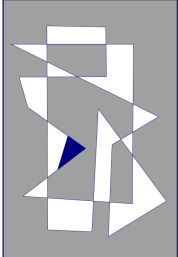|	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	Place these values in their approximate positions on the number line. (A) 3/10 of 48% (B) 1/7 (C) 45% of 1/3 0.14 0.15	Write the expression $a - (b - c)$ as an equivalent expression without parentheses. Support your answer by choosing values for a, b, and c, and evaluating the expression.	Write algebraic expressions for the measures of angles 1, 2, and 3. Explain your thinking. 1 2 3 $a°$	Maalik's family is on vacation. Using the stopwatch on his phone, Maalik finds that it takes 3 minutes and 40 seconds to travel between mile markers 47 and 51. If they leave the car on cruise control, how long will it take them to arrive at the next town 38 miles away?	You have eight coins that are identical except that one weighs less than the others. Using a balance scale, how can you find the light coin in just two weighings?
2	The prime factorization of m is $2^4 \cdot 7^2 \cdot 13^2$. The prime factorization of n is $2^3 \cdot 11^2 \cdot 13$. Use exponents to write the prime factorization of $m \cdot n$.	Whether you multiply me by 6 or add me to 6, the answer is the same. What number am I?	Draw a quadrilateral that (1) has one pair of opposite right angles and (2) is not a parallelogram. What can you say about the other two angles?	A reduced fat brand of milk claims to have 75% less fat than whole milk. The reduced fat milk contains 0.9% fat. What percentage of the whole milk is fat?	If R = 60 and T = 100, find the product of the least common multiple and the greatest common factor of R and T.
3	Which number can be made closer (but not equal) to one half: a decimal containing exactly four digits that are all different, or a fraction containing exactly four digits that are all different? Challenge yourself to use only mental math (and some estimation).	$E = a^{b^c} = a^{(b^c)}$ a, b, and c are equal to 2, 3, and 4 in some order. (All three numbers are different.) How many different values can E have? What are the two smallest values?	Draw a two-dimensional figure that has a very large perimeter and a small area. Fit it within a box about the size of the one that holds this problem.	80% of the students at a fundraiser are from Claire's school. When fourteen people from her school leave for a band concert, the percentage decreases to 62.5%. How many people are at the fundraiser now?	N is a 3-digit number abc (where a, b, and c represent the digits). When the last digit c is moved from the end to the beginning of the numeral, the value of N increases by 369. How does the value of N change when you move the beginning digit a to the right end instead?

Grade 6
Monthly Problem Set 4 Solutions

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	Values: A: 0.144 B: approx. 0.1429 C: 0.15	$a - b + c$ Example: If $a = 10$, $b = 4$ and $c = 2$ then $a - (b - c) = 10 - (4 - 2) = 8$ $a - b + c = 10 - 4 + 2 = 8$ Encourage students to choose some negative values for a, b, or c if they know how to add and subtract negative numbers!	angle 1: $180 - a°$ angle 2: $a°$ angle 3: $180 - a°$ Angles 1 and 3 both combine with the given angle to make a straight angle (180°). Angle 2 combines with angle 1 (or 3) to make a straight angle.	About 35 minutes	Put three coins on each side. If they balance, then weigh the other two coins to see which one is lighter. If they do not balance, choose any two coins from the lighter side. If they balance, the light coin is the other one. If not, choose the lighter one! (There are other methods.)
2	The prime factorization of $m \cdot n$ is $2^7 \cdot 7^2 \cdot 11^2 \cdot 13^3$. (Can you see an easy way to find this?)	1.2 $6 + 1.2 = 7.2$ $6 \cdot 1.2 = 7.2$	You can't draw rectangles, because they are parallelograms! The other two angles are *supplementary*. (They have a sum of 180°.)	3.6% If the fat content is reduced by 75%, then 25% of the original amount remains. The original amount must be 4 times greater. $0.9\% \cdot 4 = 3.6\%$	6000 $(20 \cdot 300)$ Notice that 6000 is also the product of R and T!
3	.5012 is closer to one half than $\frac{50}{98}$ is, because $\frac{50}{98}$ is $\frac{1}{98}$ greater than one half, and $\frac{1}{98} > 0.01$ (Why?). By the way, the decimal for $\frac{1}{98}$ is very cool. Try it!	*E* can have five different values. The two smallest are 65,536 and 262,144. (There would have been six values except for the fact that $2^4 = 4^2$. The two smallest values come from using $a = 4$. The two largest values, which come from $a = 2$, are enormous!)	A sample drawing:	16 people	The value of *N* increases by 306. Possible values of *N* are 145, 256, 367, 478, and 589. What patterns do you see?

Grade 6
Monthly Problem Set 5

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	If $Q = 3^4 \cdot 7^2$, then what is the prime factorization of $21 \cdot Q$? Extra: What is the prime factorization of Q^2?	Create a real-world situation to match this algebraic expression: $$C + 0.18 \cdot C$$ Extra: Rewrite it as an equivalent expression using only one operation.	A triangle's base increases by 10%, and its height decreases by 10%. By what percentage does its area increase or decrease?	Raul read 66 pages of his book today, which is about 22% more than he read yesterday. How many pages did he read yesterday?	What is the smallest three-digit *abundant* number? (A number is abundant if the sum of its factors—not including itself—is greater than the number.)
2	Insert the correct relational symbol (<, >, or =) to make the statement true: 30% of 70% ___70% of 30%	Solve the equations $\frac{1}{y} = 2$ and $\frac{1}{z} = 3$. Explain your thinking.	Draw a pentagon containing four interior right angles.	If you draw one card at random from a standard deck of 52 playing cards, what is the probability that it will be black or will not be a face card?	113 → 53 251 → 810 384 → 1596 1001 → 20 479 → ? ? → 1240
3	Jared and Annie collect marbles. Between them, they have 247 marbles. They bring some of their marbles to school to show each other. Jared brings $\frac{2}{3}$ of his marbles. Annie brings $\frac{3}{5}$ of hers. When they get to school, they discover that they have each brought the same number of marbles! Who has the larger marble collection? How much larger is it?	If $a \div b = \frac{2}{3}$, what is the value of $a \div 0.25b$? Explain your thinking.	If a 10-inch (diameter) pizza costs $8.50, what should be the cost of a 12-inch pizza?	If you choose three different numbers from the list 1, 2, 3, 4, 5, what is the probability that you will be able to form a triangle using the three numbers as side lengths?	Copy this drawing, and color it so that no region touches a region of the same color along an edge. Use the fewest colors possible.

Grade 6
Monthly Problem Set 5 Solutions

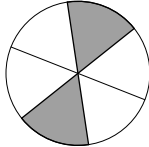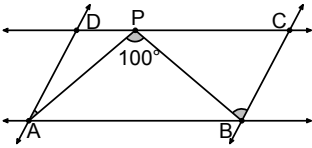|	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	$21 \cdot Q = 3^5 \cdot 7^3$ $Q^2 = 3^8 \cdot 7^4$ Notice that $21 \cdot Q$ has one additional factor both 3 and 7, because $21 = 3 \cdot 7$.	Sample situation- C: cost of a restaurant meal without the tip. Expression: the total cost of the meal with an 18% tip. Equivalent expression: $1.18 \cdot C$	The area decreases by 1%. For example, if $b = 30$ and $h = 40$, then $A = 600$, and if $b = 33$ and $h = 36$, then $A = 594$, which is 1% less than 600. How can you prove that this is true for *all* triangles?	Raul read about 54 pages.	102 1 + 2 + 3 + 6 + 17 + 34 + 51 = 114 114 is greater than 102.
2	= Both are equal to 21%. Some students may use 10% as a benchmark. (For example, 10% of 70% is 7%, etc.) Some may use pictures.	$y = \frac{1}{2}$ or 0.5 $z = \frac{1}{3}$ or $0.\overline{3}$ Some students may think that there is no solution. A sample explanation (of *many* possible): When a fraction equals 2, the numerator is always twice the denominator.	No solution: The sum of the interior angles of a pentagon is 540. 90° · 4 = 360°, and 540° − 360° = 180°, so the remaining angle would have to be straight! (The polygon would need a sixth side in order to "close up.")	$\frac{23}{26}$ or about 88.5% One method: number of blacks + number of non-face – number that are both = 40 + 26 − 20 = 46. $\frac{46}{52} = \frac{23}{26}$ Another method: 52 − number of red face cards = 52 − 6 = 46, etc.	20252 1452 (How many answers are possible?) The first part of the answer is the sum of the digits of the original number. The last part of the answer is the product of the digits of the original number.
3	Annie's collection has 13 more marbles than Jared's (130 compared to 117).	$\frac{8}{3}$ or $2\frac{2}{3}$ Multiplying the divisor by 0.25 makes it one-fourth of its original value. When the divisor becomes one-fourth as large, the quotient become four times larger (because the divisor "fits into" the dividend four times as many times).	$12.24 (not $10.20) Hint: The amount of pizza is determined by area, not length. 10 inches and 12 inches are length measurements, not area. Some students may suggest a small reduction in price from $12.24 to encourage customers to buy a larger pizza.	The probability is 30%. There are 10 ways to choose 3 numbers from the list. 3 of these (**bold**) can form a triangle. 1, 2, 3 1, 2, 4 1, 2, 5 1, 3, 4 1, 3, 5 1, 4, 5 **2, 3, 4** 2, 3, 5 **2, 4, 5** **3, 4, 5** Can you find a fast way to tell which sets of numbers can form a triangle?	Here is a way to do it with three colors. Is it possible to do it with two? How carefully must you copy the drawing?

Grade 6
Monthly Problem Set 6

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	The greatest common factor of *a* and *b* is 14. The least common multiple of *a* and *b* is 420. Find the values of *a* and *b*.	Which expression does not belong? Why? $$\frac{a \cdot b}{c} \qquad \frac{a}{b \cdot c} \qquad a \cdot \frac{b}{c}$$ $$\frac{a}{c \div b} \qquad \frac{a}{c} \cdot b$$	Estimate the area and perimeter.	The speed of light is approximately 3×10^8 meters per second. A typical person's hair grows about ½ inch per month. Use a ratio to compare speed of light to the speed of hair growth. (Use a calculator as needed!)	Explain the error in this thinking process: $$\frac{2 \cdot a + b}{a + b} = \frac{2 \cdot a + \cancel{b}}{a + \cancel{b}}$$ $$= \frac{2 \cdot a}{a} = \frac{2 \cdot \cancel{a}}{\cancel{a}} = 2$$
2	Write a fraction/whole number division equation for this picture. Then write and answer an "equal sharing" story problem for your equation and the picture. The large rectangle represents one whole.	For *n = 1, 2, 3, 4, 5*, and *6* graph the number of "missing circles", *M* (output), vs the picture number, *n* (input), and find a formula.	A prism has right-triangular bases with side lengths of 3 cm, 4 cm, and 5 cm. (1) If the volume of the prism is 576 cm³, what is its surface area? (2) If you shrink the prism to create a similar prism with a volume of 9 cm³, what is the new surface area?	PQST is a parallelogram. The length PQ is $\frac{4}{5}$ of QR. (1) Use a ratio to compare the areas of PQST and triangle QRS. (2) Suppose that the area of trapezoid PRST is 65 cm². What can you say about QR and *h*?	Use a compass and a protractor to draw an accurate enlargement of this design. Draw more copies, joining them to create a figure with rotational symmetry.
3	Write the exact solution to this equation in decimal and fraction form. Do not use a calculator. $$\frac{1}{3} \cdot 0.375 = 0.\overline{1} \cdot N$$	Find a simpler algebraic expression that always gives the same answer as $$\frac{4 \cdot x^2 - 1}{2 \cdot x - 1}$$ (except when $x = \frac{1}{2}$).	The points are the centers of the three regular nonagons. What is the measure of the marked angle?	If you roll two standard 6-sided dice, what is the probability that one number will be a factor of the other?	If the cube root of N is between 400 and 900, what whole numbers must the square root of N be between? Use as much mental math as you can! Try not to use a calculator.

Grade 6
Monthly Problem Set 6 Solutions

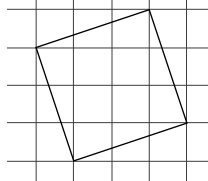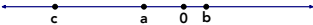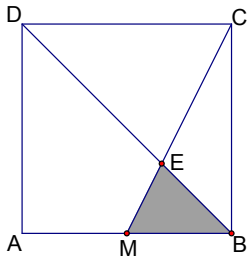|	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	There are four solutions. Encourage students to find as many as they can and to look for patterns! a = 14 b = 420 a = 28 b = 210 a = 42 b = 140 a = 84 b = 70	The best answer is probably $$\frac{a}{b \cdot c}$$ because it is not *equivalent* to the others. (It does not always give the same answer as the others. The one with the division symbol does!)	Area: approximately 15.5 square units. Perimeter: a little less than 19 units. Ask students to discuss their strategies!	The speed of light is about 6×10^{16} (or 60 quadrillion) times faster than the rate at which human hair grows. Or comparing the other way: your hair grows at about one 60 quadrillionth of the speed of light.	It does not make sense to "cross out" b, because this means subtracting the same number from the numerator and denominator, which changes the value of the fraction. (Actually, if you substitute numbers for a and b, the answer will *never* be 2 unless b = 0!)
2	Equation: $\frac{2}{3} \div 4 = \frac{1}{6}$ Sample story: After a race, four runners share $\frac{2}{3}$ of a gallon of water equally. How many gallons does each racer get? ($\frac{1}{6}$ of a gallon)	 Possible formulas: $M = n \cdot (n-1) \div 2$ $M = (n^2 - n) \div 2$	Surface area: 1164 cm^2 New surface area: 72.75 cm^2 Notes: - The height of the original prism was 96 cm. - The side lengths of the bases of the new prism are 0.75, 1, and 1.25 ($\frac{1}{4}$ as long as the original side lengths.)	(1) The area of PQST is 1.6 times the area of QRS, or the area of QRS is $\frac{5}{8}$ of the area of PQST. (2) QR and h can have many values, but their product must always equal 50.	 Are other answers possible?
3	Note: Be sure that students notice the repeating bar over the 1. 1.125 $\frac{9}{8}$ Ideas: Think of one or both decimals as fractions. (Since $\frac{1}{3} = 0.\overline{3}...$, what is a fraction for $0.\overline{1}...$?)	$2 \cdot x + 1$ Why doesn't it give the same answer when $x = \frac{1}{2}$?	160° Students will probably need to find measures of the nonagons' interior angles at some point. Would the nonagons form a "ring" if you continued the pattern? If so, how many of them would it take? Why?	$\frac{11}{18}$ or about 61% There are 36 outcomes in the sample space. 22 of them (in bold) are favorable. **1,1** **1,2** **1,3** **1,4** **1,5** **1,6** **2,1** **2,2** 2,3 **2,4** 2,5 **2,6** **3,1** 3,2 **3,3** 3,4 3,5 **3,6** **4,1** **4,2** 4,3 **4,4** 4,5 4,6 **5,1** 5,2 5,3 5,4 **5,5** 6,5 **6,1** **6,2** **6,3** 6,4 6,5 **6,6**	Between 8,000 and 27,000

Grade 6
Monthly Problem Set 7

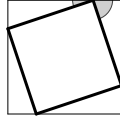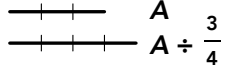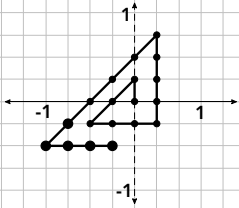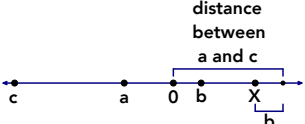|	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	Solve for x. $$\frac{1}{1+\dfrac{3}{1+x}}=\frac{8}{29}$$ (Suggestion: Experiment with fractions and decimals if you like, but it may easier to think of x as a fraction.)	Estelle has finished 16 of the M questions on her test. How many more questions must she answer in order to complete $\frac{1}{3}$ of the test? Show two ways to write the algebraic expression: (1) Multiply, then subtract. (2) Subtract, then divide.	The area of the circle is 81π units^2. It is divided into 6 equal parts. What is the perimeter of the shaded region in terms of π?	A spinner has three colors: red, blue, and yellow. The red and blue sectors are the same size. If you spin twice, the probability that one spin will be red and the other will be blue is 8%. What is the measure of the yellow sector's central angle?	Which two of these expressions are equal? How do you know? $5^{2000} \quad 3^{3000} \quad 2^{4500}$ $6^{1500} \quad 8^{1500} \quad 4^{2500}$ Bonus: Order the expressions from least to greatest.
2	Let $X=0.2\overline{3}$. Write 10% of X as a fraction in simplest form.	C is an even number, and D is an odd number. For what possible values of C and D will the expression $$C^2 - C \cdot D + D^2$$ be an even number? Explain your reasoning.	The largest faces of three identical rectangular prisms having whole number side lengths are joined to create a new rectangular prism whose volume is 10,455 cm^3. What are the dimensions of the new prism? How much surface area is lost when they are joined?	A faucet drips once every 5 seconds into a bathtub of (inner) dimensions L: 54", W:23", and D:15". A typical drop of water has a volume of about 0.25 milliliters. About how long does it take to fill the tub?	The nine factors of 36 are arranged in a 3 by 3 grid so that every row and column has the same product. What is this product?
3	In this problem, you explore what happens when you add the same number to a numerator and a denominator. Suppose that p, q, and n are all natural numbers. Then $$\frac{p+n}{q+n}$$ is always between _____ and _____.	Find one or more algebraic expressions that show a shortcut for calculating the sum of N consecutive whole numbers beginning with 30.	ABCD is a parallelogram. What is the relationship between $\angle$DAP and $\angle$CBP? Explain your thinking.	Madar is driving from his house to the store and from there to work. He has traveled 75% of the way to the store, which is 40% of his total travel distance. Which of the two parts of his trip is longer? Use a ratio to compare the lengths of the two parts.	Can the multiplicative inverse (reciprocal) of a real number ever equal the additive inverse (opposite) of the number? If so, give an example. If not, explain why not.

Grade 6
Monthly Problem Set 7 Solutions

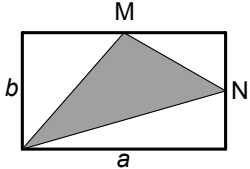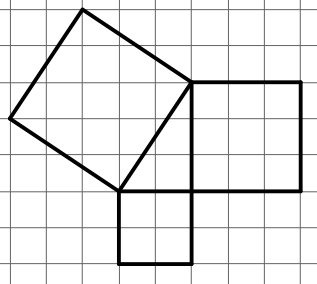|	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	$x = \dfrac{1}{7}$ Hint: a and $\dfrac{1}{a}$ are reciprocals of *each other* (even when a is not a whole number)	(1) $\frac{1}{3} \cdot M - 1$ (2) $(M - 48) \div 3$ What will happen if Estelle has already answered more than $\frac{1}{3}$ of the questions? Will the expressions give you any useful information?	$6\pi + 36$ units	216°	$2^{4500} = 8^{1500}$ To see why, imagine writing out all 4500 2s and putting them into groups of 3. Bonus: (1) 6^{1500} (2) 8^{1500} and 2^{4500} (3) 5^{2000} (4) 3^{3000} and (5) 4^{2500} Hint: Arrange the factors of each number in 500 identical groups.
2	$\dfrac{7}{300}$ There are *many* strategies: (1) Begin with $10 \cdot X$. (2) Write the decimal over 10 and create equivalent fractions. (3) Compare to the decimal for $\frac{1}{3}$, etc.	The expression will not represent an even number for any whole number values of C and D. C^2 and $C \cdot D$ are always even. D^2 is always odd. even − even + odd = odd	New dimensions: 15 cm by 17 cm by 41 cm. Lost surface area: 2788 cm^2 Four faces are "lost" when the three prisms are joined. $17 \cdot 41 \cdot 4 = 2788$ cm^2	About 71 days It would actually take longer, because this calculation does not take evaporation into account.	216 If you haven't already, try to create the 3 by 3 grid!
3	$\dfrac{p + n}{q + n}$ is between $\frac{p}{q}$ and 1. Must p, q, and n be natural numbers for this to be true? Does it matter whether $p < q$ or $p > q$?	There are many correct expressions. Examples: $\left(30 + \dfrac{N-1}{2}\right) \cdot N$ $\left(\dfrac{60 + N - 1}{2}\right) \cdot N$ $30 \cdot N + \dfrac{N(N-1)}{2}$ What is the sum of N consecutive whole numbers starting with A?	∠DAP and ∠CBP have sum of 100°.	The trip from Madar's house to the store is a little longer $(1\frac{1}{7}$ times as long) than his trip from the store to work. Or: His trip from the store to work is $\frac{7}{8}$ as long as the trip from his house to the store. Can you compare the two parts of his trip using a difference instead of a ratio?	No. A real number and its reciprocal always have the same sign (positive or negative), because their product equals 1. Opposites always have opposite signs (except for 0, which does not have a reciprocal). Note: There is actually a pair of *imaginary* numbers that works (i and $-i$).

Grade 6
Monthly Problem Set 8

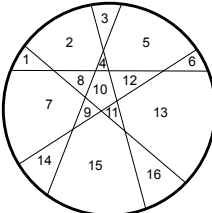|	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	Find the prime factorization of $6^5 \cdot 14^3 \cdot 9$. Write your answer in exponential form.	Suppose that $$a \; \Delta \; b = \sqrt{a+b}$$ $$a \; \dagger \; b = (a-b)^2$$ Solve this equation. $$2 \; \Delta \; (3 \; \dagger \; x) = 1\tfrac{1}{2}$$	This is a rhombus. Explain why it must also be a square.	Gail, who is 4'9'' tall, is standing next to her older sister in a photo. In the photo, Gail is 2.5 inches tall and her sister is $3\tfrac{1}{16}$" tall. How tall is her sister?	The price of a coat jumps 60% when winter arrives. By what percentage must the price drop in order to return to its original value? (Be careful! The answer is not 60%.)
2	Draw a line segment. Name its length A. Then draw a segment with a length of $A \div \tfrac{3}{4}$. Make a real-world story to fit your picture and the expression $A \div \tfrac{3}{4}$.	Show at least four ways to use a calculator to find the decimal for $$\frac{3 \cdot 7}{5 \cdot 11}$$ using only the numbers, 3, 5, 7, and 11 and the keys x, ÷, and =. (No parentheses)	Design a home aquarium with a volume of about 1.6 ft^3 that uses between 1000 1100 square inches of glass. The top of the aquarium is not made of glass. (Be sure that your measurements are realistic for an aquarium. Feel free to use a calculator.)	The probability of drawing a red marble from a bag is $\tfrac{1}{4}$. The probability of drawing red, replacing it, and then drawing yellow is $\tfrac{1}{8}$. Three of the marbles in the bag are neither red nor yellow. How many marbles are in the bag?	What is the sum of the first 100 digits in this pattern? Explain your thinking. 2221222112222111222221111...
3	What are the next five ordered pairs in this pattern? (Read the pairs across then down—the way you read a book.) (0, 0) (0 ,0.25) (-0.25, 0) (-0.5, -0.25) (-0.25, -0.25) (0, -0.25) (0.25, -0.25) (0.25, 0) (0.25, 0.25) (0.25, 0.5) (0.25, 0.75) (0, 0.5) (-0.25, 0.25) (-0.5, 0) Hint: Plot the points!	X = − (c − a) − b Describe or show the location of X on the number line as accurately as you can. Explain. Bonus: Would the location of X change if you moved '0' to a different place on the line but left a, b, and c where they are?	M is the *midpoint* of $\overline{AB}$. Find or estimate the ratio of the area of triangle EMB to the area of square ABCD.	A snail travels 30 mm in straight line. It moves 6 mm per minute for the first half of the distance. Then it gets tired and travels 4 mm per minute the rest of the way. A second snail travels the same distance in the same amount of time, moving at a constant rate. How fast did the second snail travel?	Chiron picked a number, multiplied it by a, and then subtracted b. When he started with the same number and multiplied it by b and then subtracted a, he got the same answer. What can you say about the number that he started with?

Grade 6
Monthly Problem Set 8 Solutions

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	$2^8 \cdot 3^7 \cdot 7^3$ How can you find this answer without first writing the number in standard form?	$x = 2\frac{1}{2}$	Hint: You can use marked angles to explain why the rhombus' angles *must* be 90°. 	Her sister is about 5'10" tall.	The price must drop by 37.5%. (It does not matter what the original price was.)

2	A
	 $A \div \frac{3}{4}$
	
	Challenge students not to use a specific number for the length, just the meaning of division by a fraction. Students should create a story making $A \div \frac{3}{4}$ be the number that A is $\frac{3}{4}$ of.
		Sample answers:
		$3 \cdot 7 \div 5 \div 11$
		$3 \div 5 \cdot 7 \div 11$
		$3 \div 11 \cdot 7 \div 5$
		$7 \div 5 \div 11 \cdot 3$
		In each answer, you must start with or multiply by 3 and 7, and you must divide by 5 and by 11. You may do these things in any order.
			There are *many* possible answers. Some possibilities:
			
			W: 20" D: 11" H: 13"
			  glass: 1026 in^2
			  volume: 1.66 ft^3
			W: 24" D: 9" H: 13"
			  Glass: 1074 in^2
			  Volume: 1.625 ft^3
				12
				
				Hints: $\frac{1}{4}$ of the marbles in the bag must be red. In order for the probability of 'red then yellow' to be $\frac{1}{8}$, half of the marbles in the bag must must be yellow.
					155
					The pattern has:
					  1 two and 0 ones
					  2 twos and 1 one
					  3 twos and 2 ones, etc.
					There are many other patterns that students may use, including some that involve square numbers and the sequence 2, 5, 8, 11, 14…

3	(-0.75, -0.25) (-1, -0.5) (-0.75, -0.5) (-0.5, -0.5) (-0.25, -0.5) The pairs make a right-triangular spiral. 	 Challenge students to use the meanings of *subtraction* and *opposite* instead of using numbers. Bonus: Yes, X would move twice as far and in the same direction as you move 0.)	The area of triangle EMB is $\frac{1}{12}$ of the area of square ABCD. The key fact is that (using MB as the base) the height of the triangle is $\frac{1}{3}$ of a side length of the square Students may estimate this. Some may be able to explain *why* (perhaps using similar triangles).	4.8 (or $4\frac{4}{5}$) millimeters per minute Why does it make sense that answer is *not* the average of 4 and 6? How could you have predicted it would be closer to 4 than to 6?	Chiron started with the number -1.

Grade 6
Monthly Problem Set 9

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	Two years ago, the volume of water in a reservoir dropped by 20%. Last year, it dropped by 50% of what was left. By what percentage of its current amount must the volume of the reservoir increase in order to return to its original volume?	Write an algebraic equation for T in terms of G and F. G F T 10 10 14.4 10 20 18.8 10 30 23.2 20 10 24.4 20 20 28.8 20 30 33.2 Extra: If T = 23.68, what are G and F?	Lizzie builds a 3D shape by joining congruent cubical blocks face-to-face. The blocks have whole number side lengths greater than 1 cm. The volume of the shape is 8918 cm^3, and its surface area is 2646 cm^2. Describe or draw the shape. Include the important measurements.	Oji runs $\frac{2}{3}$ as far as Arpita. Grace runs $\frac{4}{5}$ as far as Arpita. How far does Oji run compared to Grace?	Draw 5 lines through a circle to create a many regions as possible. What is the greatest possible number of regions?
2	a and b have a difference of 15. What can you say about the greatest common factor (GCF) of a and b? What happens if the difference of a and b is a prime number?	Calculate using mental math. Explain your thinking. Name the properties that you used. $$\frac{864(0.8)-864(0.3)}{0.36(20)+0.64(20)}$$	The shorter segments are sides of a regular 16-gon. Calculate the measure of $\angle x$. 	An 1108 in^3 kitchen sink fills in 151 seconds. A 7.4 ft^3 tub fills in $9\frac{1}{4}$ minutes. How many times faster is the tub faucet's flow rate compared to that of the kitchen faucet?	You have $1.15 in coins, but you cannot make change for a dollar, half-dollar, quarter, dime, or nickel. What coins do you have (and how many of each)?
3	Use each digit 7, 8, and 9 (and no others) exactly one time per number to make as many mixed numbers and improper fractions as you can. Order them from least to greatest. Extra: What are the two closest different numbers in the list?	A formula for the *period* of a pendulum (the time for one complete oscillation) is $$T = 2\pi\sqrt{\frac{L}{g}}.$$ T is the period (secs), L is the length of the pendulum (ft), and g is the acceleration caused by Earth's gravity (ft per sec per sec). Using a 9-inch pendulum, you measure a period of 0.96 seconds. Calculate the acceleration caused by Earth's gravity.	M and N are midpoints of the sides of the rectangle. Write an algebraic formula for the area (A) of the shaded triangle in terms of the lengths, a and b, of the rectangle's sides. 	Jessica spins a spinner four times hoping to land on red four times in a row. She repeats this experiment many times. If she hits all four reds in about 13% of her experiments, approximately what fraction of the spinner is red?	Show a way to cut and rearrange the two smaller squares to make the large square.

Grade 6
Monthly Problem Set 9 Solutions

	Number and Operation (NO)	Algebra (A)	Geometry and Measurement (GM)	Data and Probability; Ratios and Rates (DR)	General (G)
1	150% After the volume decreases by 20%, 80% remains. When this decreases by 50%, 40% of the original amount remains. The volume needs to increase by 60% of the *original* amount, which is 150% of the current amount of 40% (1.5 times as much).	$T = G + 0.44 \cdot F$ *Note*: This formula is from basketball. T stands for True Shooting Attempts (TSA); G represents Field Goals Attempted (FGA); and F represents Free Throws Attempted (FTA). Extra: $G = 14$; $F = 22$ Are other answers possible?	The shape is a cube with three blocks per side and one *corner* block missing. Each block is 7 cm on a side. The "almost cube" contains 26 blocks, and the lengths of its (full) sides are 21 cm. *Hint*: It may help to start by finding the prime factorization of 8918.	Oji runs $\frac{5}{6}$ as far as Grace. Suggest that students draw a picture. See if they can think of splitting it into 15 equal parts. (A few students may discover the idea of dividing: $\frac{2}{3} \div \frac{4}{5} = \frac{5}{6}$. Ask them why it makes sense to do this!)	You can make at most 16 regions. Notice the star! Can you make 22 regions with six cuts? Try it!
2	The GCF of a and b must be one of the numbers 1, 3, 5, or 15 (a factor of 15). If the difference is a prime number, then a and b will have only two possible GCFs: 1 or that number.	21.6 Use the distributive property twice to get $$\frac{864(0.5)}{1(20)} = \frac{432}{20} = \frac{216}{10}$$ $$= 21.6$$	33.75° Interior 16-gon angle: 180° • (16 − 2) / 16 = 157.5° 157.5° • 3 = 472.5° Interior pentagon angle sum: 180 • (5 − 2) = 540° (540° − 472.5°)/2 = 33.75°	The tub faucet's flow rate is about 3.14 times faster than the kitchen faucet's flow rate. (It is a complete coincidence that this number is approximately equal to π!)	You have a half-dollar, a quarter, and four dimes. (This is the only possible answer. How can you be sure?)
3	From least to greatest: $7\frac{8}{9}$, $7\frac{9}{8}$, $\frac{78}{9}$, $8\frac{7}{9}$, $8\frac{9}{7}$, $\frac{87}{9}$, $\frac{79}{8} = 9\frac{7}{8}$, $9\frac{8}{7}$, $\frac{97}{8}$, $\frac{89}{7}$, $\frac{98}{7}$ The two closest numbers are $\frac{79}{8}$ and $9\frac{7}{8}$, because they are equal, but the two closest *different* numbers are $\frac{78}{9}$ and $8\frac{7}{9}$ (with a difference of $\frac{1}{9}$).	About 32 ft/sec/sec* Most students will guess and test values for g. Some may think backwards: (1) Divide T (0.96 sec) by 2π (2) Square the result (3) Take the reciprocal (4) Multiply by L (0.75 ft) *A falling objects' speed increases by about 32 ft per second every second.	$A = \frac{3}{8} \cdot a \cdot b$ or $A = \frac{3}{8}ab$ Areas Rectangle: ab Upper-left triangle: $\frac{1}{4}ab$ Lower triangle: $\frac{1}{4}ab$ Upper-right triangle: $\frac{1}{8}ab$ Shaded triangle: $ab - \frac{1}{4}ab - \frac{1}{4}ab - \frac{1}{8}ab = \frac{3}{8}ab$	Approximately 3/5 Imagine splitting the spinner into 5 equal parts and coloring 3 of them red. Count each sector as a separate outcome. There are 5×5×5×5 = 625 outcomes in the sample space and 3×3×3×3 = 81 outcomes in the event. $81 \div 625 = 0.1296 \approx 13\%$	There are *many* ways! *Hint*: Consider repositioning the smaller square first.