Deep Math Projects
Magic Numbers
Introduction

Topics

- Addition and subtraction of whole numbers

What students should know

- Add and subtract whole numbers of up to three digits.

How the activity extends math standards

- Use complex addition and subtraction patterns to
    - Solve challenging problems.
    - Make and test predictions.
    - Justify conclusions.
- Explore patterns involving addition and subtraction of even and odd numbers.
- Generalize whole number addition and subtraction patterns to other numbers.

Getting started

Display or hand out a copy of the Opener without saying anything about it. Start an open-ended discussion about it by asking students what they notice and what they wonder. Do not hand out or discuss the directions yet.

The Teacher's Guide has examples of student responses to help you guide the discussion. These are only suggestions. It is not important to discuss all of them. The ideas should come mainly from your students. Your job is to acknowledge and help them help clarify their ideas. Once they figure out the basics of how the puzzle works, they are ready to begin the project.

If you are not available to lead this discussion, ask your students to work in a small group to discuss and write down the things that they notice and wonder. You may check back with them later to ensure that they are ready to begin the project.

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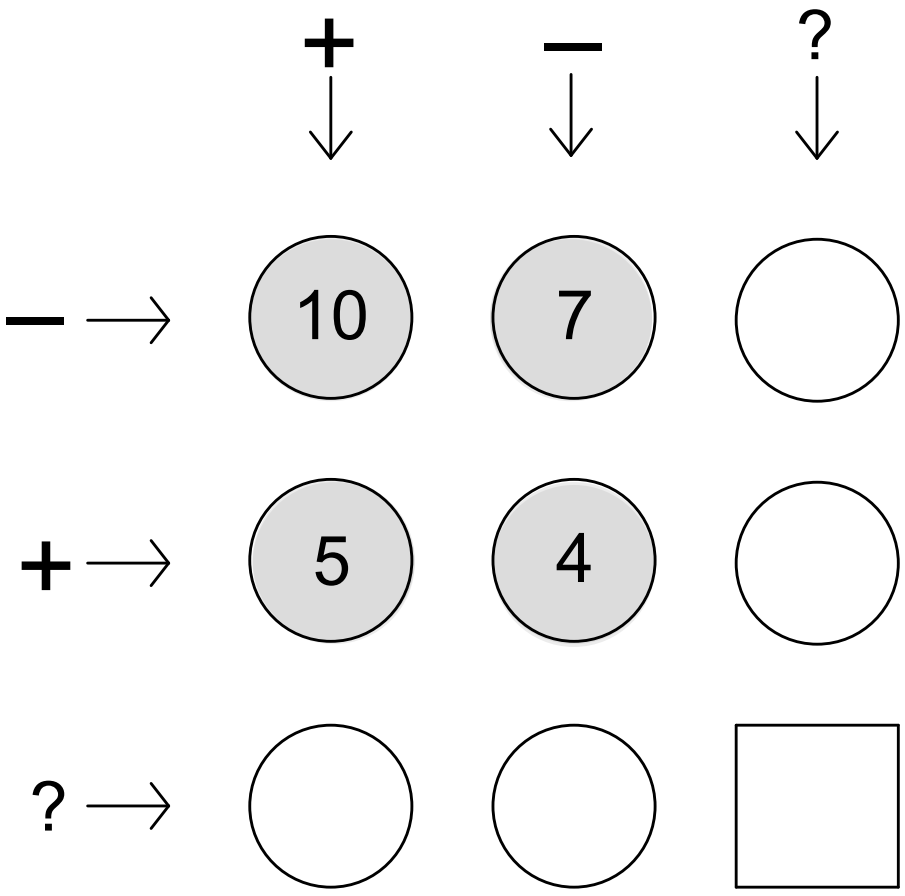
Magic Numbers
Opener



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Magic Numbers
Directions

PART 1 (Use Handout 1.)

1. Complete Puzzle #1.

2. Explain why the number in the square is called a *magic number*.

3. Complete Puzzles #2, #3, and #4 so that all of them have the same magic number as Puzzle #1.

4. What strategies did you use? Explain your thinking.

PART 2 (Use Handout 2.)

5. Add 1 to each shaded number in Puzzle #1. What happens to the magic number? Explain your thinking.

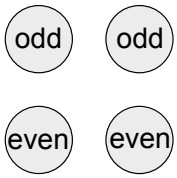
6. Predict what will happen if you increase each shaded number by 50. Explain your thinking.

7. Use the other three blank puzzles to test your prediction.

PART 3 (Use another copy of Handout 2.)

8. Create four puzzles that have odd magic numbers.

9. When the shaded circles look like this, the magic number is always even:



   How will the shaded circles look when the magic number is odd?

10. Explain why.

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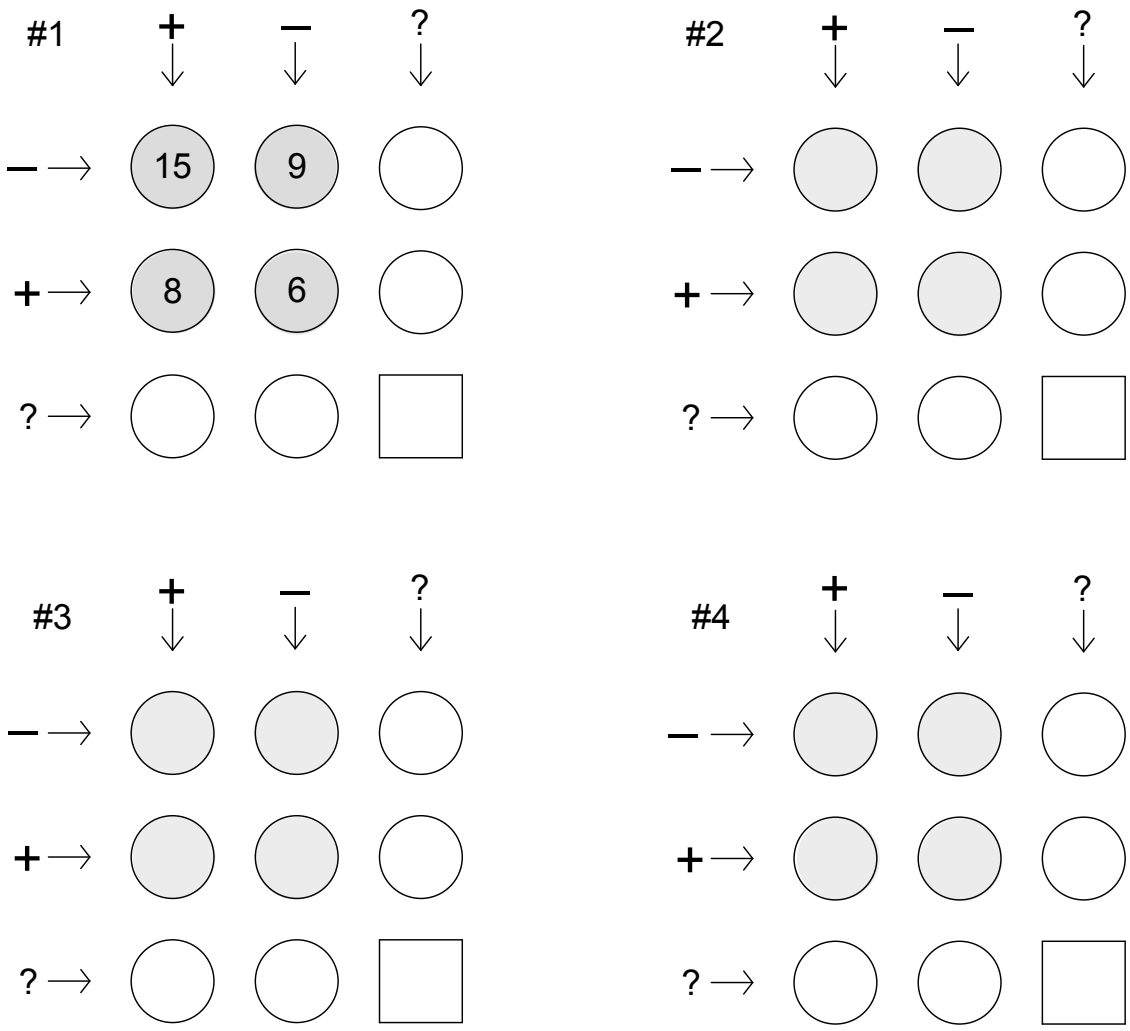
Magic Numbers
Handout 1



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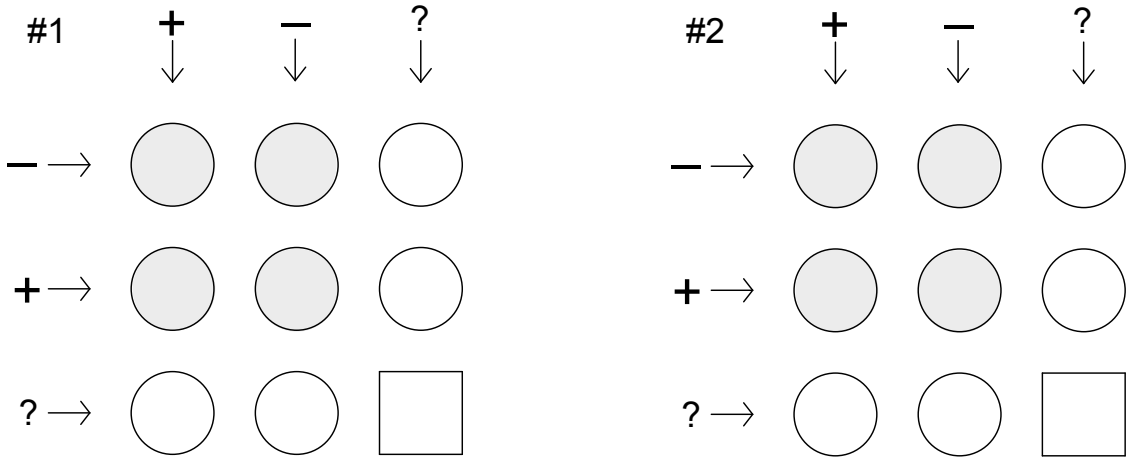
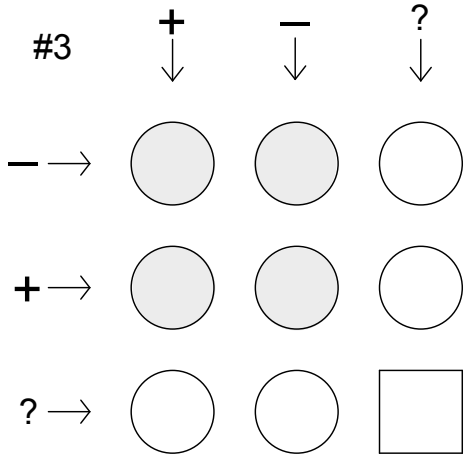
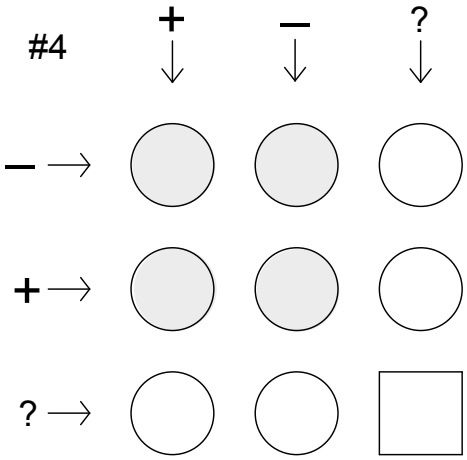
Magic Numbers
Handout 2







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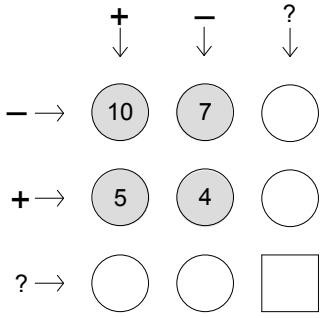
Magic Numbers
Teacher's Guide

Encourage mindful learning by guiding students to *observe* (notice) and *question* (wonder) before, during, and after the project.

Before starting the problems

Show students the opener and ask them what they notice and what they wonder.



Accept all responses without labeling them "right" or "wrong." Do not give directions yet.

Examples

- *I notice* three rows and three columns.
- *I notice* that all of the shapes are circles except for one square.
- *I wonder* if the square is important?
- *I wonder* if we are supposed to put numbers inside the empty shapes?
- *I notice* that the "+" and "–" symbols alternate.
- *I wonder* if the question marks stand for operations (like "+" or "–")?
- *I notice* that the numbers get smaller (if you read them like a book).

Some students may be able to complete the opener correctly with no instructions. If so, they may notice and wonder further.

- *I notice* that I can replace the question marks by operations.
- *I notice* that the number in the square is both a sum (vertically) and a difference (horizontally) of numbers in the circles.
- *I wonder* if this will still happen if I put other numbers the shaded circles?

Once students have finished sharing their observations and questions, distribute the directions and Handout 1. They are now ready to being work on the project! If they create their own interesting questions while "wondering," you may follow up on them later or even make them part of the investigation!

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While solving the problems

Students continue noticing and wondering as they solve the problem: whether working independently or collaboratively, when making progress or when they are stuck, while thinking or calculating.

Examples

- *I wonder* what happens when all four shaded numbers are the same?
- *I wonder* what happens when all four shaded numbers are odd?
- *I wonder* if I can make the magic number equal 0?
- *I notice* that the magic number pattern always seems to work.
- *I wonder* if the pattern still works when the numbers are very large?
- *I wonder why* the pattern always works?
- *I notice* that I can predict what happens when I make small changes to the shaded numbers.
- *I notice* that when all of the shaded numbers are even, the magic number is also even.

After solving the problems

Noticing now involves revisiting and summarizing ideas. *Wondering* may include clarifying things that are still confusing, thinking of new questions to ask, or coming up with new ways to extend to problem.

Examples

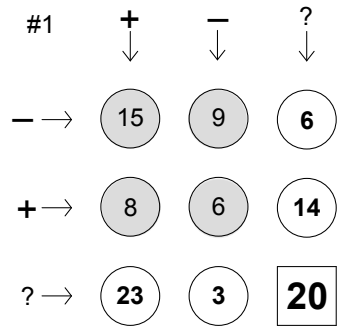
- *I noticed* that I could predict how a sum or difference would change when I changed the other numbers.
- *I noticed* that there are patterns to how addition and subtraction work together.
- *I noticed* that these patterns continued for all kinds of numbers that I tried.
- *I noticed* that some problems have an infinite number of solutions!
- *I noticed* that I could save work by spotting patterns and making predictions.

- *I wonder* what happens when I put the smaller numbers on the top row?
- *I wonder* if the pattern still works when some of the numbers are negative?
- *I wonder* what will happen if I keep making the shaded numbers smaller and smaller until they turn negative?
- *I wonder* if the pattern still works when some of the numbers are fractions or decimals?
- *I wonder* if there is a quick way to predict the magic number without filling in the whole puzzle?

Magic Numbers
Notes and Solutions

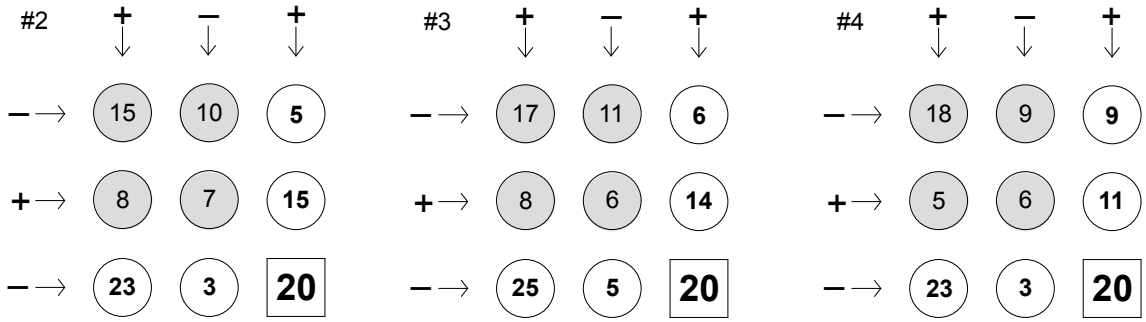
Part 1

1.



2. The number in the square is called a *magic number* because the sum of the numbers on the right equals the difference of the numbers in the bottom. (The "?" on top stands for "+," and the "?" on the left stands for "–.")

3. Sample solutions:



4. Sample strategies:
   All strategies compare the new solution to the original solution.
   For #2: Add 1 to the shaded numbers on the right.
   For #3: Add 2 to the shaded numbers at the top.
   For #4: Add 3 to the top-left number; subtract 3 from the bottom-left number.

   There are many strategies and solutions! Help your students to notice what happens to the sum when they change the addends (or to the difference when they change the minuend or subtrahend). For example, adding the same number to the minuend and subtrahend leaves the difference unchanged.

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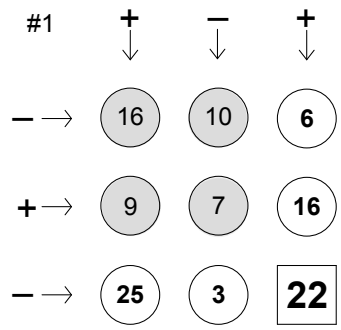
Part 2

5. The magic number increases by 2. For example:

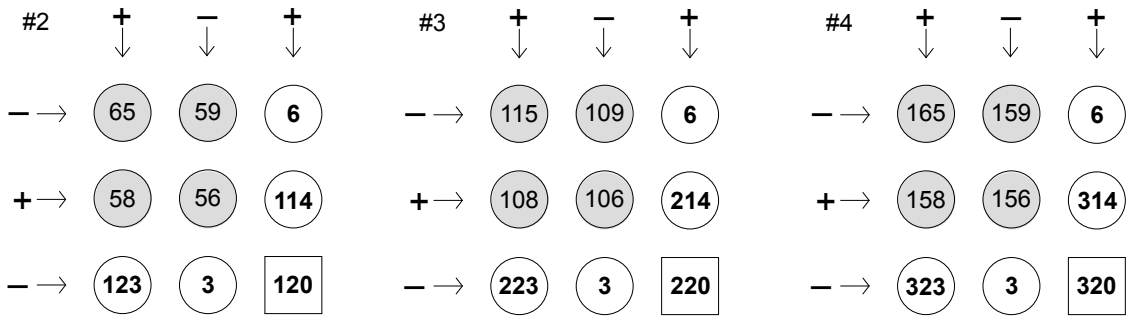


A good explanation: Create more examples and notice that the sum always increases by 2. A better explanation: Notice details and explain *why*. For example, observe that the two differences always remain the same, while the two sums each increase by 2.

6. Students' predictions will vary.

The magic number will increase by 100. (Imagine increasing each shaded number by 1 fifty times. The magic number will increase by 2 fifty times.)

7. In these examples, 50 is added three times to the shaded numbers in the original Puzzle #1. Each time, the magic number increases by 100. Students may use other examples to test their predictions.



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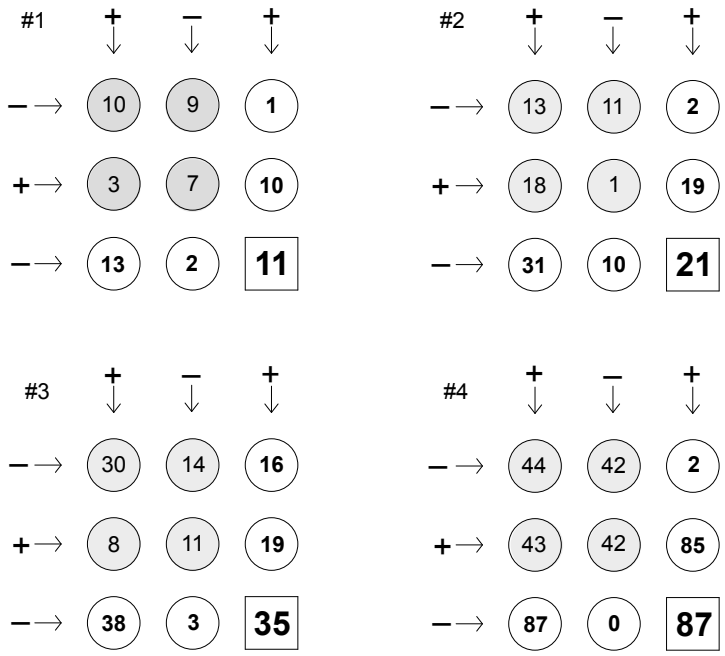
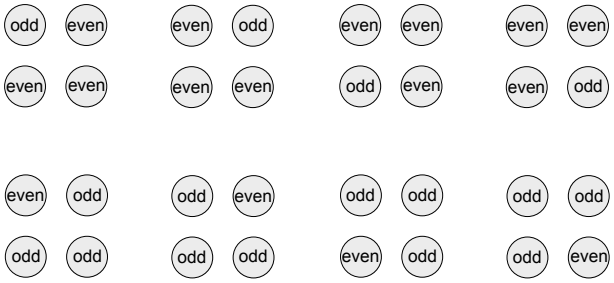
Part 3

8. Sample solutions:



9. There are eight possibilities. (Your students may notice just two or three.)



Each possibility has either 3 evens and 1 odd or 1 even and 3 odds. All other combinations make the magic number even.

10. Some students may use these observations to support their explanation:

even + even = even
odd + even = odd
even + odd = odd
odd + odd = even

Some may even be able to explain why these relationships are true. (Hint: An even number is a whole number of pairs. And odd number is a whole number of pairs plus one.)

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