Hopping Home
Introduction

Topics
- Meanings and properties of multiplication and division
- Strategies for multiplying and dividing multi-digit numbers (and possibly simple fractions)
- Relationships between multiplication and division

Materials
- Red and blue pencils or markers (for drawing arrows between circles)

What students should know
- Understand what multiplication and division mean.
- Multiply and divide numbers within a 10 by 10 multiplication table.
- Have some experience with “frames and arrows” problems (recommended).

How the activity extends math standards
- *Create* strategies for multiplying and dividing multi-digit numbers and simple fractions.
- *Discover* properties that connect multiplication and division.
- *Explore* division expressions in which the divisor is greater than the dividend.
- *Analyze* complex patterns.
- *Solve* problems with multiple solutions. *Recognize* when all solutions have been found.

Getting started
Display or hand out copies of the Opener. Start an open-ended discussion by asking students what they *notice* and *wonder*. Use the “I notice/I wonder” T-chart if you like. Give students plenty of time to get comfortable sharing math ideas openly.

For your information: the main idea is for students to “hop home” from 64 to 2 in exactly five jumps—using only “x 4” and ÷ 8.” In order to make patterns easier to see, have students draw blue arrows between the circles for “x 4” and red arrows for “÷ 8.”

The *Noticing and Wondering: Sample Responses* page has ideas to help you guide discussion. Remember that these are only suggestions. The ideas should come mainly from your students. It may be easier for students to notice than to wonder.

If you are not available to lead the opening discussion, ask your students to work in small groups to discuss and write down the things that they notice and wonder. You may check back with them later to help them identify questions to explore.

Once your students understand the problem and have questions to explore, they may begin. They may use copies of the Recording Page to show their answers.

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Opener

$\times 4$ $\div 8$

64					2
64					2
64					2
64					2

Name ________________________________

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I notice	I wonder

Name ______________________________

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Recording Page

x 4 ÷ 8

64					2
64					2
64					2
64					2

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Noticing and Wondering: Sample Responses

Note: Students may have many ideas that are not on this page.

1. Before starting the problem

I notice that this looks like a *frames and arrows* problem.
The task is to turn 64 into 2 in exactly five steps using only "x 4" and "÷ 8."
I notice that there are four copies of the problem.
I notice that the two operations have different colors.
I notice that each problem has five steps.
I wonder if it is possible to do the problem in more or fewer steps.
I notice that I have to make the number 64 a lot smaller.
I notice that some of the numbers in the circles could be in hundreds or thousands.
I wonder if the problem has a solution.
I wonder if I could do the problem backwards and turn 2 into 64 (still using x 4 and ÷ 8).

2. While solving the problem

I notice that many of the numbers in the circles (in different problems) are the same.
I notice that it helps to break numbers apart when I divide large numbers.
I notice that multiplying by 4 and then dividing by 8 does the same thing as dividing by 2.
I notice that dividing by 8 and then multiplying by 4 also does the same thing as dividing by 2.
I wonder if I can divide a smaller number by a larger number.
I notice that some of the numbers in the circles can be fractions.
I notice a connection between the answers to 8 ÷ 4 and 4 ÷ 8.
I notice that I need to divide by 8 more often than I multiply by 4 (to make 64 get smaller).
I notice that there is more than one solution.
I wonder how many solutions there are.
I wonder how I can tell when I have found all of the solutions.
I notice a pattern in the number of arrows of each color.

3. After solving the problem

I wonder why every solution has 3 reds and 2 blues.
I notice that multiplying by 4 is like multiplying by 2 twice.
I wonder what dividing by 8 is like. (It is like dividing by 2 three times.)
I notice that I can turn 64 into 2 by dividing by 2 five times.
I notice that multiplying by 2 and dividing by 2 undo each other.
I wonder if I can make my own frames and arrows problems that have a lot of solutions.
I wonder if there are patterns to the answers when I do a problem backwards.
I wonder if I could turn 64 into 2 by multiplying and dividing by other numbers (like x 3, ÷ 6).
If so, *I wonder* how many steps it would take.

The process of asking and answering new questions can go on for a long time.
You never stop noticing and wondering!

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Solutions and Math Notes

Solutions

There are ten solutions. Those listed first are often discovered first. Students should be encouraged to find as many solutions as they can. Searching for *all* of them is an excellent challenge for exceptionally curious, persistent, or advanced students.

These solutions show color codes instead of arrows. D8: Divide by 8. M4: Multiply by 4.

64	8	1	4	16	2	(D8, D8, M4, M4, D8)
64	8	32	4	16	2	(D8, M4, D8, M4, D8)
64	8	32	128	16	2	(D8, M4, M4, D8, D8)
64	256	1024	128	16	2	(M4, M4, D8, D8, D8)
64	256	32	128	16	2	(M4, D8, M4, D8, D8)
64	256	32	4	$\frac{1}{2}$	2	(M4, D8, D8, D8, M4)
64	256	32	4	16	2	(M4, D8, D8, M4, D8)
64	8	32	4	$\frac{1}{2}$	2	(D8, M4, D8, D8, M4)
64	8	1	4	$\frac{1}{2}$	2	(D8, D8, M4, D8, M4)
64	8	1	$\frac{1}{8}$	$\frac{1}{2}$	2	(D8, D8, D8, M4 M4)

The problem gradually becomes more challenging as students find more solutions. (See the next page for ideas about calculation strategies.) Encourage them to stick with the problem for a long time and not to shy away from large numbers and fractions.

After students have found a few solutions, ask them to figure out what their solutions have in common. They may eventually discover that all of their solutions contain exactly two blue arrows and three red arrows. In other words:

Every solution contains exactly two M4s and three D8s!

This discovery may help students find more (or even all) solutions. You have found all of them when you have listed every possible order for writing down two blue arrows and three red arrows. (It may help to try to list them in an organized way.)

Multiplication and division strategies with multi-digit numbers

This problem offers wonderful learning opportunities for students who have *not* been taught methods for multiplying or dividing multi-digit numbers, because they need to develop their own strategies.

For example, they may draw diagrams showing groups. Or they may break numbers into smaller parts, multiply or divide each part, and then recombine the parts (which gives them a preview of the *distributive property*—something they will learn much more about in later grades.) Be sure to ask students to share and compare their strategies!

Example:

1024 ÷ 8
Decompose 1024 as 800 + 80 + 80 + 64
If you split 800 into 8 equal groups, each group has 100.
If you split 80 into 8 equal groups, each group has 10.
If you split 64 into 8 equal groups, each group has 8.
Recombine: 100 + 10 + 10 + 8 = 128

Multiplication and division strategies with fractions

Students may be able to create strategies for multiplying and dividing with simple fractions. They do not need to learn rules or shortcuts for this!

$4 \div 8 = \frac{1}{2}$	Students may think of dividing 4 into 8 equal parts.
$\frac{1}{2} \times 4 = 2$	Students may think of 4 groups of $\frac{1}{2}$ (or $\frac{1}{2}$ groups of 4).
$\frac{1}{8} \times 4 = \frac{1}{2}$	Students may think of 4 groups of $\frac{1}{8}$ (or $\frac{1}{8}$ groups of 4).

Multiplication and division properties

Students who pay close attention may discover some properties of multiplication and division. A few examples:

- Multiplying by 4 and then dividing by 8 does the same thing as dividing by 2.
- Dividing by 8 and then multiplying by 4 also does the same thing as dividing by 2.
- Multiplying by 4 twice does the same thing as multiplying by 16.
- The order in which you do “x 4” and “÷ 8” doesn’t matter.

You might ask your students to compare expressions like $8 \div 4$ and $4 \div 8$. Since they have probably never seen a situation where the second number is greater than the first, ask them to think about this. For example:

- 8 ÷ 4 means to divide 8 into 4 equal parts. You get answer of 2.
- 4 ÷ 8 means to divide 4 into 8 equal parts. You get an answer of $\frac{1}{2}$.

Notice that the denominator of the (simplified) quotient of 4 ÷ 8 is the same as the quotient of 8 ÷ 4. Students make test other division expressions to see if this always happens.

Reasons for the properties

Since 2 is quite a bit smaller than 64, you need to divide by 8 more often than you multiply by 4. With some help and plenty of time to think and discuss, some students may be able to dig a little deeper into this idea as suggested below.

Many of the same numbers keep appearing in the circles. Try listing them in order.

$$\frac{1}{8} \quad \frac{1}{2} \quad 2 \quad 4 \quad 8 \quad 16 \quad 32 \quad 64 \quad 128 \quad 256 \quad 1024$$

There is *almost* a pattern here, but there are a few numbers missing. Can you find them?

Most of the time, each number is twice the one to its left. You can make the pattern work *all* of the time by filling in three more numbers:

$\frac{1}{4}$ (between $\frac{1}{8}$ and $\frac{1}{2}$)
1 (between $\frac{1}{2}$ and 2)
512 (between 256 and 1024)

The new list looks like this.

$$\frac{1}{8} \quad \frac{1}{4} \quad \frac{1}{2} \quad 1 \quad 2 \quad 4 \quad 8 \quad 16 \quad 32 \quad 64 \quad 128 \quad 256 \quad 512 \quad 1024$$

Now notice:

1 jump to the right multiplies by 2.	1 jump to the left divides by 2.
2 jumps to the right multiplies by 4.	2 jumps to the left divides by 4.
3 jumps to the right multiplies by 8.	**3 jumps to the left divides by 8.**
etc.	

A combination of jumps (example)

Multiplying by 4 and then dividing by 8:
Jumps 2 to the right and then 3 to the left.
Net effect: Jumps 1 to the left (Divides by 2.)

Connecting to the problem

To get from 64 to 2, you need to jump 5 to the left (divide by 2 five times).
Think about how multiplying by 4 two times and dividing by 8 three times (in any order) always does this!

Note: The ideas on this page relate to the concept of *exponents*.