Deep Math Projects
Area Challenge
Introduction

Topics

- Area
- Names and attributes of polygons
- Perimeter

What students should know

- Recognize and draw polygons
- Classify polygons using their attributes
- The meaning of perimeter

How the activity extends math standards

- Create a variety of polygons having a given area.
- Explore areas of polygons with fractional square units.
- Explore perimeters involving diagonal segments on a grid.
- Explore relationships between area and perimeter.

Getting started

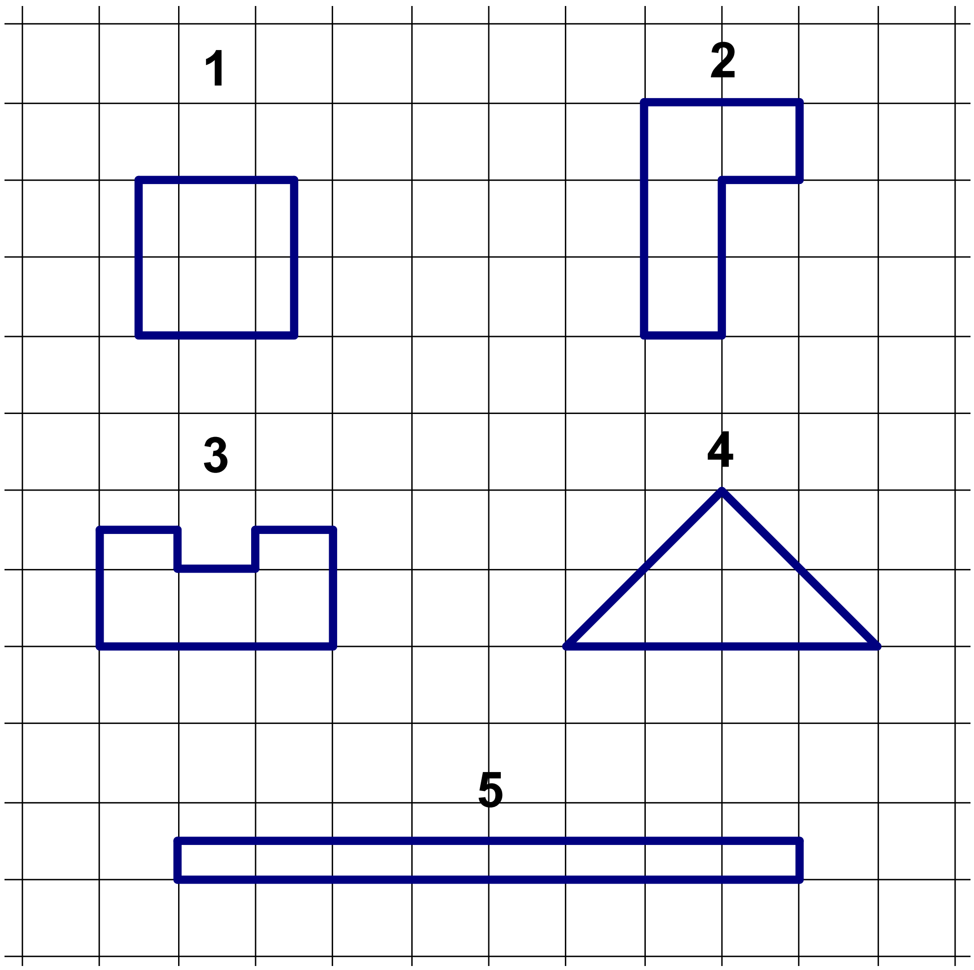
Display or hand out a copy of the Opener without saying anything about it. Ask students what they notice and what they wonder. (Do not hand out or discuss the directions yet.)

The Teacher's Guide has samples of student responses. These are suggestions. The ideas should come mainly from your students. Your job is to acknowledge their ideas and to help clarify them as needed. Once students recognize that each polygon contains 4 square units, they are ready to begin the project. (However, the discussion should also include ideas about perimeter and properties of polygons.)

If you are not available to lead this discussion, have your students write down the things that they notice and wonder and discuss these in a small group. You may check back with them later to ensure that they are ready to begin.

Area Challenge
Opener

What do you notice? What do you wonder?



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Area Challenge
Directions

PART 1

1. Draw at least ten different polygons that have areas of 4 square units.

2. How do you know that your shapes are polygons and that their areas are 4 square units? Talk or write about your ideas.

3. Do any of your polygons remind you of something in the real world or your imagination?

PART 2

4. Name each polygon in as many mathematical ways as you can. Talk about your reasons.

5. Find or estimate the perimeters of your polygons. Talk about how you figured them out.

PART 3

6. Draw at least four triangles that have areas of 4 square units. Compare and contrast your answers. Explain your thinking.

7. Order your triangles from the least to the greatest perimeter. Explain your thinking.

ANOTHER CHALLENGE:

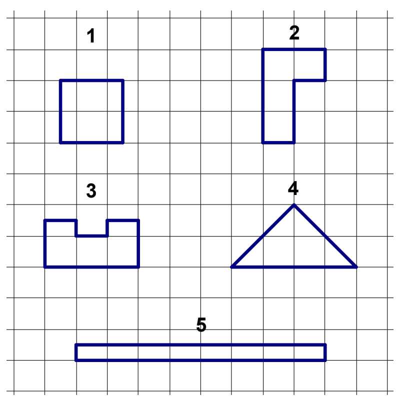
8. Draw a 4-square-unit polygon, making the perimeter as great as you can. Explain your thinking.

Area Challenge
Teacher's Guide

Guide students to *notice* and *wonder* before, during, and after the activity.

Before solving the problems

Show students the opener and ask them what they *notice* and what they *wonder*. Use the conversation to stimulate students' interest, clarify concepts, and discuss vocabulary needed to understand the problem (for example: the meanings of *polygon*, *area*, *and perimeter*). If students create new questions while "wondering," try to follow up on them later—or even make them part of the investigation!



Accept all responses without naming them "right" or "wrong." Allow students to question and clarify each other's ideas and to agree or disagree and explain why. Sample responses:

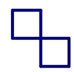
- *I notice* one triangle, two quadrilaterals, a hexagon, and an octagon.
- *I notice* that the vertical sides of the square don't line up with the grid.
- *I notice* that the triangle is the only shape with diagonal sides.
- *I notice* that all of the polygons look symmetrical except for #2.
- *I wonder* if all five polygons have something in common.
- *I wonder* if all of the polygons have the same perimeter.
- *I notice* that polygon #2 has 4 whole squares inside of it. (Its *area* is 4 sq. units.)
- *I notice* that the other four polygons have parts of squares inside of them.
- *I notice* that all of the partial squares are half-squares.
- *I notice* that if you break apart and rearrange the parts of the squares, each polygon has 4 whole squares inside of it.

After you finish discussing students' observations and questions, distribute the directions and some graph paper. They are ready to begin!

While solving the problems

Students continue "noticing" and "wondering." Samples:

- *I notice* that my polygon looks like a ….
- *I wonder* how many polygons of area 4 I can make using only whole squares.
- *I notice* that I can find a new solution by cutting off a piece of an old solution and putting it somewhere else.
- *I notice* that a lot of my polygons have an even number of sides.
- *I wonder* if it is harder to make solutions with an odd number of sides.
- *I wonder* if it counts as a new polygon when I flip or turn a polygon.
- *I wonder* if I can use smaller pieces of grid squares to make polygons.
- *I wonder* if I this is a polygon.



- *I notice* that I can use the side my grid paper to measure diagonal lengths.
- *I wonder* why it is so hard to make a lot of different triangles with the same area?
- *I wonder* what is the most sides my polygon can have.

After solving the problems

Notice: summarize important ideas. Wonder: clarify things that are still confusing, think of new questions to ask, or come up with ways to extend to problem.

What did you *notice* when you solved this problem? What did you learn?

- *I noticed* that I could make 5 polygons from whole squares.
- *I noticed* that area is not always length times width.
- *I notice* that the perimeters can be different even when the areas are the same.
- *I noticed* that I could build my polygon from small pieces.
- *I noticed* that the diagonal inside a square is longer than the sides.
- *I noticed* that some triangles are hard to break into easier shapes.
- *I notice* that my diagonal lengths were not whole numbers.

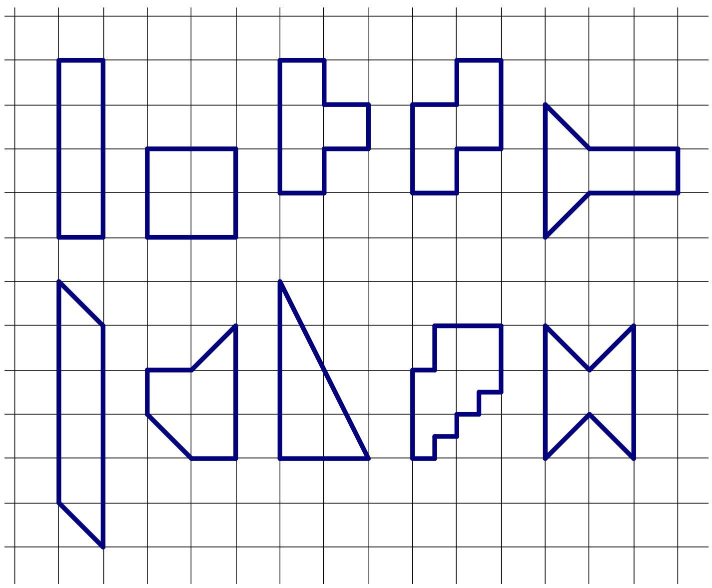
What do you still *wonder*? What new questions could you ask?

- *I wonder* if areas can be different when perimeters are the same?
- *I wonder* how you could figure out areas of shapes made with curves?
- *I wonder* if it’s always possible to make a shape with any area and perimeter.

Area Challenge
Math Notes and Solutions

Part 1

1. Sample solutions: (Students need at least ten polygons.)



2. Answers will vary. Students should use the definition of a polygon to explain why their shapes are polygons. To explain why the areas are 4 square units, they may talk about counting whole squares or partial squares, breaking the polygons into pieces and rearranging the parts, etc. For triangles like the one above (in the middle of the bottom row), they may imagine putting two of them together to make a rectangle with an area of 8 and taking half of it.

3. Answers will vary. This is mainly for fun and to get the thinking creatively.

Suggestions: Encourage students to share and compare their polygons. Ask them to explain their thinking and justify their answers to each other.

Many students may stop at Part 1 or simply continue creating more polygons with the same area. Those who need further challenge may continue.

Part 2

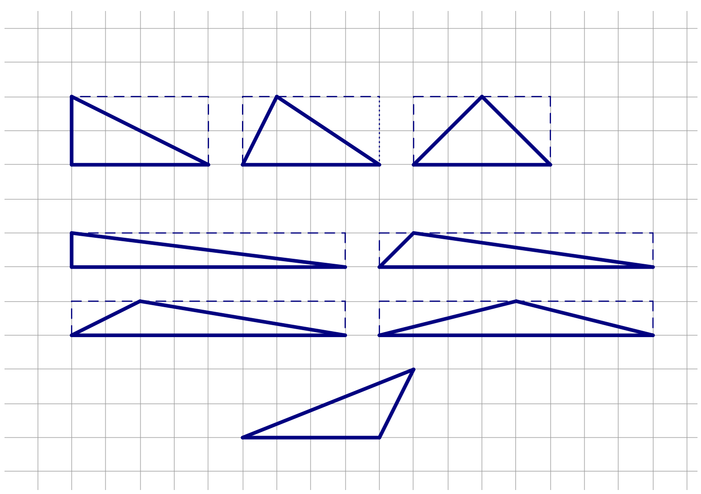
4. Students will name most of the quadrilaterals by their number of sides: triangle, quadrangle, pentagon, etc. For special shapes, they should give more names. For example, a rectangle is also a parallelogram because its opposite sides are parallel. And a square is also parallelogram and a rectangle, because its opposite sides are parallel and it has all right angles.

5. Students' answers will vary according to the polygons that they create. Diagonal sides will usually not have whole number lengths. Students may estimate them or measure them (approximately) with the edge of a piece of grid paper.

Part 3

Note: Part 3 is very challenging. For most students, Parts 1 and 2 are probably enough.

6. Sample answers: (Students are asked to find at least four.)



A common strategy is to think about each triangle as half of a rectangle that has an area of 8 square units. Once they have found one triangle, they may notice that they can slide a vertex (shown on the top of each triangle) along the edge of the rectangle.

Students may not find a triangle like the one on the bottom, because the "half of a rectangle" idea does not work for it. It would be a great challenge for kids even to figure out why the area is 4! (Hint: Surround the triangle with a rectangle, but this time, take away the areas of the extra parts.)

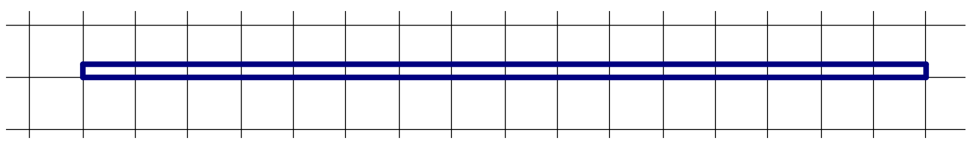
7. The perimeters of the top three triangles are close. They get just a little smaller from left to right (as the vertex on the top moves toward the middle).

   Approximate perimeters of the top triangles:

   Left: Almost 10 and a half
   Middle: A little less than 10
   Right: A little greater than 9 and a half

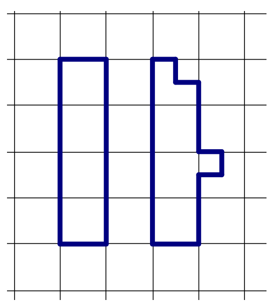
   The perimeters of the skinny triangles are much greater than the ones on top (between about 16 and 17), They also get smaller as the top vertex moves to the middle.

8. One strategy is to make a very long and skinny rectangle. Polygon 5 on the opening page is an example of this. The longer and skinnier you make it, the greater the perimeter becomes. For example, students could draw a rectangle that is 16 units long and ¼ of a unit wide!



   The perimeter of this rectangle is 32 and a half units! It might be very hard to draw, but students could just explain what they are doing instead of trying to draw it perfectly.

   Some students might try to build polygons with lots of little parts that stick out. For example, they could start with a simple rectangle, and take out a little piece and place it somewhere where it sticks out.



   They could then repeat this process over and over. (It makes the perimeter even greater if you take the little pieces out of the middles of the sides instead of the corners.

   No matter which strategy they use, there is actually no limit to how great the perimeter can be.